Unit - I Basic Concepts & Isentropic flows

Gas dynamics - It deals with the study of motion of gases & its effects.

Compressible flow:

The density does not remains constant, It varies with pressure and temperature.

$$\boxed{\text{Density}, \rho \neq \text{Constant}}$$

(Examples) Gases and vapours.

Incompressible flow:

The density of fluid remains constant.

$$\boxed{\text{Density}, \rho = \text{Constant}}$$

Examples: Liquids.

Stagnation State and its Properties:

Stagnation state:

Stagnation state of a gas or vapours is defined as the "state of a gas is obtained by "decelerating a gas isentropically to zero velocity at zero elevation"."

Stagnation Enthalpy (h_0):

Stagnation enthalpy of gas or vapour is defined as, when it is adiabatically or isentropically decelerated to zero velocity at zero elevation.

Obtaining of stagnation enthalpy equation,

Put,

$$\left.\begin{aligned} h_1 &= h \\ C_1 &= C \end{aligned}\right\} \text{for initial state.}$$

Consider the gas is decelerated to zero velocity at zero elevation.

$$\left.\begin{aligned} C_2 &= C \\ h_2 &= h_0 \end{aligned}\right\} \text{For final State}$$

$$\therefore \frac{C_1^2}{2} + h = h_0.$$

$$\Rightarrow \boxed{h_0 = h + \frac{C^2}{2}}$$

where h_0 - stagnation enthalpy
h - static enthalpy
C - Fluid velocity.

Stagnation Temperature (T_0):

Stagnation temperature of the gas or vapour is defined as the temperature of a gas when it is adiabatically (or) isentropically decelerated to zero velocity at zero elevation.

For a perfect gas, this is defined through stagnation enthalpy.

WKT, Stagnation Enthalpy (h_0) $= h + \frac{C^2}{2}$ ($\because h = C_p T$ for perfect gas)

$$\Rightarrow C_p T_0 = C_p T + \frac{C^2}{2} \qquad h_0 = C_p T_0.$$

$\div C_p$ on both sides,

$$T_0 = T + \frac{C^2}{2} \cdot \frac{1}{C_p}.$$

$$\boxed{\text{Stagnation Temperature } (T_0) = T + \frac{C^2}{2C_p}}$$

Derivation of ratio b/w stagnation temperature (T_0) and static temperature (T)

WKT, Stagnation temperature $T_0 = T + \frac{c^2}{2C_p}$

$\div T$ on both sides

$$\Rightarrow \frac{T_0}{T} = \frac{T}{T} + \frac{c^2}{2C_pT}$$

$$\frac{T_0}{T} = 1 + \frac{c^2}{2C_pT}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{c^2}{2\left[\frac{\gamma R}{\gamma - 1}\right]T}$$

For perfect gas,

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$\frac{T_0}{T} = 1 + \frac{c^2}{2\frac{(a^2)}{\gamma - 1}}$$

$\therefore a = \sqrt{\gamma RT}$

$a^2 = \gamma RT$.

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\left(\frac{c^2}{a^2}\right)$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} m^2$$

$\because$ Mach number $(m) = \frac{c}{a}$

$$M^2 = \frac{c^2}{a^2}$$

$$\boxed{\therefore \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2}$$

Where T_0 – Stagnation temp, T – Static temp

M – Mach number.

Stagnation Pressure (P_0):

Stagnation Pressure of the gas is defined as the pressure obtained when the gas is isentropically decelerated to zero velocity at zero elevation.

For an isentropic flow process,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$$

For Stagnation Condition,

Put $P_1 = P$, $P_2 = P_0$.

$T_1 = T$, $T_2 = T_0$

Therefore $\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$

$$\boxed{\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\gamma/\gamma-1}} \quad \left(\because \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2\right)$$

P_0 - Stagnation Pressure

P - Static Pressure

M - Mach number

Stagnation density (ρ_0):

Stagnation density is the density of the gas when its is adiabatically or isentropically decelerated to zero velocity ~~to~~ at zero elevation.

For isentropic flow Process,

$$\frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

For stagnation conditions.

$\rho_1 = \rho$, $P_1 = P$

$\rho_2 = \rho_0$, and $P_2 = P_0$.

$$\frac{\rho_0}{\rho} = \left(\frac{P_0}{P}\right)^{1/\gamma}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\cancel{\gamma}}{\gamma - 1} \times \frac{1}{\cancel{\gamma}}}$$

$$\boxed{\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/\gamma - 1}}$$

where

ρ_0 - Stagnation density

ρ - ~~st~~ Static density

M - Mach number

Stagnation Velocity of Sound (a_0)

WKT,

Velocity of sound $a = \sqrt{\gamma R T}$

For Stagnation Condition,

Put $a = a_0$; $T = T_0$.

Therefore $\Rightarrow$ $a_0 = \sqrt{\gamma R T_0}$

Where $R \rightarrow$ Gas constant.

$$R = \frac{\gamma - 1}{\gamma} \times C_p$$

$$a_0 = \sqrt{\cancel{\gamma} \times \frac{\gamma - 1}{\cancel{\gamma}} \times C_p \times T_0}$$

$$= \sqrt{(\gamma - 1) C_p \cdot T_0}$$

$$a_0 = \sqrt{(\gamma - 1)\, h_0}$$

Where a_0 - Stagnation Velocity

h_0 - Stagnation Enthalpy.

Steady Flow Energy Equation:

Total Energy entering the system = Total Energy leaving the system

$$gz_1 + \frac{1}{2}c_1^2 + U_1 + P_1V_1 + Q = gZ_2 + \frac{1}{2}c_2^2 + V_2 + P_2V_2 + W$$

$\therefore$ $h = U + PV$

$$\therefore\ gz_1 + \frac{c_1^2}{2} + h_1 + Q = gz_2 + \frac{c_2^2}{2} + h_2 + W$$

The above eqn is known as a general form of the steady flow energy equation (SFEE).

For Turbomachines, such as turbines and Compressor, there is no heat transfer takes place and change in Potential energy is negligible.

$\therefore$ Heat transfer $Q = 0$

$z_1 - z_2 = 0.$

$$\boxed{\therefore\ \frac{c_1^2}{2} + h_1 = \frac{c_2^2}{2} + h_2 + W.}$$

For nozzle & diffuser:

$Q = 0, \quad W = 0, \quad gz_1 = gz_2$

The SFEE becomes,

$$\boxed{\frac{C_1^2}{2} + h_1 = \frac{C_2^2}{2} + h_2}$$

Velocity of sound: (or) local velocity of sound (a)

The velocity of sound with which sound waves progate in a medium is called "velocity of sound(a)".

The velocity of a sound is given by,

$$\boxed{a = \sqrt{\gamma RT}}$$

Mach Number

It is the ratio b/w inertia force & elastic force.

It is denoted by an index of "M".

$$M^2 = \frac{\text{Inertia force}}{\text{Elastic force}} = \frac{\rho \cancel{A} c^2}{K \cancel{A}} = \frac{\rho c^2}{K}$$

$$M^2 = \frac{\rho c^2}{\rho a^2} \qquad (K = \text{Bulk modulus} = \rho a^2)$$

$$M^2 = \frac{c^2}{a^2} \Rightarrow \boxed{M = \frac{c}{a}}$$

Mach number is defined as the Ratio of the fluid velocity (C) to the velocity of sound (a)

$$M = \frac{C}{a} \qquad \text{WKT} \quad a = \sqrt{\gamma RT}$$

$$\Rightarrow \boxed{M = \frac{C}{\sqrt{\gamma RT}}}$$

Mach Cone:

Tangents drawn from the Point of Source "S" on the spheres define a Conical surface referred to as "Mach Cone".

Mach angle:

The angle b/w the mach line & the direction of the body is called "Mach angle".

$$\text{Mach angle } \alpha = \sin^{-1}\left(\frac{1}{M}\right)$$

Mach waves: (or) Mach lines:

The lines at which the Pressure difference is concentrated and which generate the cone are called "mach waves or Mach lines".

Effect of Mach Number on Compressibility:

Let us assumed the flow is incompressible,

WKT, Stagnation Pressure for incompressible flow is (from Bernoulli eqn)

$$P_0 = P + \frac{1}{2}\rho c^2$$

$$\frac{P_0 - P}{\frac{1}{2}\rho c^2} = 1$$

The value of the pressure coefficient C_p is unity.

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$$

$$\Rightarrow \frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1}$$

By Taylor series:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Here $x = \frac{\gamma-1}{2}M^2$ and $n = \frac{\gamma}{\gamma-1}$

$$\therefore \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} = 1 + \frac{\gamma}{\gamma-1}\left(\frac{\gamma-1}{2}M^2\right) + \frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma}{\gamma-1}-1\right)}{2!} \times \left(\frac{\gamma-1}{2}M^2\right)^2$$

$$+ \frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma}{\gamma-1}-1\right)\left(\frac{\gamma}{\gamma-1}-2\right)}{3!} \times \left(\frac{\gamma-1}{2}M^2\right)^3$$

$$= 1+\frac{\gamma}{2}M^2+\frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma-(\gamma-1)}{\gamma-1}\right)}{2}\times\left(\frac{(\gamma-1)^2}{4}M^4\right)$$

$$+\frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma-(\gamma-1)}{\gamma-1}\right)\left(\frac{\gamma-2(\gamma-1)}{\gamma-1}\right)}{6}\times\frac{(\gamma-1)^3}{8}M^6$$

$$= 1+\frac{\gamma}{2}M^2+\frac{\frac{\gamma}{\gamma-1}\left(\frac{1}{\gamma-1}\right)}{2}\times\left[\frac{(\gamma-1)^2}{4}M^4\right]$$

$$+\frac{\frac{\gamma}{\gamma-1}\left(\frac{1}{\gamma-1}\right)\left(\frac{2-\gamma}{\gamma-1}\right)}{6}\times\left[\frac{(\gamma-1)^3}{8}\times M^6\right]+\dots$$

$$= 1+\frac{\gamma}{2}M^2+\frac{\frac{\gamma}{\gamma-1}\times\frac{1}{\gamma-1}\times(\gamma-1)^2}{8}M^4$$

$$+\frac{\frac{\gamma}{\gamma-1}\times\frac{1}{\gamma-1}\times\frac{2-\gamma}{\gamma-1}\times(\gamma-1)^3}{48}M^6+\dots$$

$$= 1+\frac{\gamma}{2}M^2+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+\dots$$

$$\therefore \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = 1+\frac{\gamma}{2}M^2+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+\dots$$

Substitute the above eqn in $\frac{P_0}{P}$, we get,

$$\frac{P_0}{P} = 1+\frac{\gamma}{2}M^2+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+\dots$$

$$\frac{P_0}{P}-1 = \frac{\gamma}{2}M^2+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+\dots$$

$$\frac{P_0-P}{P} = \frac{\gamma}{2}M^2\left(1+\frac{1}{4}M^2+\left(\frac{2-\gamma}{24}\right)M^4+\dots\right)$$

$$\boxed{\frac{P_0-P}{P\left(\frac{\gamma}{2}M^2\right)} = 1+\frac{1}{4}M^2+\frac{2-\gamma}{24}M^4+\dots} \rightarrow (1)$$

WKT,

Mach number $M = \frac{c}{a}$

$$M^2 = \frac{c^2}{a^2} \qquad \therefore a = \sqrt{\gamma RT}$$

$$M^2 = \frac{c^2}{\gamma RT} \qquad a^2 = \gamma RT$$

Multiply by $\frac{\gamma}{2}$ on both sides

$$\frac{\gamma}{2} M^2 = \frac{\gamma}{2} \times \frac{c^2}{\gamma R T}$$

$$\Rightarrow \frac{\gamma}{2} M^2 = \frac{c^2}{\gamma R T}$$

Multiply P on both sides.

$$\boxed{P \times \frac{\gamma}{2} M^2 = \frac{P c^2}{\gamma R T}}$$

WKT

$$P v = R T$$

$$P = \frac{R T}{v}$$

$$P = \rho R T \quad \left(\because \rho = \frac{1}{v}\right)$$

$$P \times \frac{\gamma}{2} M^2 = \rho R \cancel{T} \left(\frac{c^2}{\gamma R \cancel{T}}\right)$$

$$P \times \frac{\gamma}{2} M^2 = \frac{\rho c^2}{2}$$

Substitute the above value in eqn (1)

$$\frac{P_0 - P}{\left(\frac{\rho c^2}{2}\right)} = 1 + \frac{M^2}{4} + \frac{2-\gamma}{24} M^2 + \cdots$$

Substitute $\gamma = 1.4$,

$$\therefore \frac{P_0 - P}{\left(\frac{\rho c^2}{2}\right)} = 1 + \frac{M^2}{4} + \frac{M^4}{40}$$

The above equation known as the pressure Coefficient equation for compressible flow.

Isentropic flow through variable ducts - Nozzles and diffusers:

Problem - 1

1. An air jet at 400 K has sonic velocity. Determine velocity of sound at 400 K, velocity of sound at stagnation condition, maximum velocity of jet, stagnation enthalpy.

Given:

Temp $T_1 = 400 K$.

To find:

(i) velocity of sound (a) (ii) velocity of sound at stagnation (a_0), (iii) Max. velocity (C_{max})

(iv) stagnation enthalpy (h_0)

Soln.

At sonic velocity condition,

Mach number $= (M) = 1$

Assume γ and R for air.

$\gamma = 1.4$, $R = 287 \ J/kgK$.

$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 400}$

$\boxed{a_1 = 400.9 m/s}$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\frac{T_0}{400} = 1 + \frac{1.4 - 1}{2} (1)^2$$

$T_0 = 400 \times 1.2 \Rightarrow \boxed{T_0 = 480 K}$

$a_0 = \sqrt{\gamma R T_0}$

$= \sqrt{1.4 \times 287 \times 480}$

$\boxed{a_0 = 439.16 \ m/s}$

$$h_0 = \frac{a_1^2}{\gamma-1} + \frac{1}{2}C^2 = \frac{1}{2}C_{max}^2 = \frac{a_0^2}{\gamma-1}$$

$$\frac{1}{2}C_{max}^2 = \frac{a_0^2}{\gamma-1}$$

$$C_{max} = \frac{2a_0^2}{\gamma-1}$$

$$C_{max}^2 = \frac{2\times 439.16^2}{1.4-1}$$

$$C_{max}^2 = 964307.53$$

$$\boxed{C_{max} = 981.99\ m/s}$$

$$h_0 = \frac{1}{2}C_{max}^2$$

$$= \frac{1}{2}(981.99)^2$$

$$\boxed{h_0 = 482.152\times10^3\ J/kg}$$

Q. Air flows through a nozzle which has inlet area of $0.001 m^2$. If the air has a velocity of 80 m/s, a temperature of 301 K and a pressure of 700 bar at the inlet section and a pressure of 250 bar at the exit, find the mass flow rate through the nozzle and assuming one dimensional isentropic flow, the velocity at the exit section of the nozzle.

Given data:

Inlet area $A_1 = 0.001 m^2$

Inlet velocity $C_1 = 80 m/s$

Inlet temp $T_1 = 301 K$

Inlet Pressure $P_1 = 700$ bar $= 700 \times 10^5 N/m^2$

Exit Pressure $P_2 = 200$ bar $= 200 \times 10^5 N/m^2$.

To find:

(i) Mass flow rate (m)

(ii) Velocity at exit (C_2)

Soln:

$$m = \rho A C = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$$

$$m = \rho_1 A_1 C_1$$

$$m = \frac{P_1}{RT_1} A_1 C_1$$

$\because PV = mRT$, $\frac{P}{RT} = \frac{m}{V} = \rho$, $\left(\rho_1 = \frac{P_1}{RT_1}\right)$

For air $\gamma = 1.4$, $R = 287 J/kgK$.

$$m = \frac{700 \times 10^5}{287 \times 301} \times 0.001 \times 80$$

$\Rightarrow \boxed{m = 64.83 kg/s}$

$$M_2 = \frac{C_2}{a_2}$$

M_2 — Find from this equation,

$$\frac{P_2}{P_1} = \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\gamma/\gamma-1}$$

$$M_1 = \frac{C_1}{a_1}, \quad a_1 = \sqrt{\gamma R T_1}$$

$$= \sqrt{1.4 \times 287 \times 301}$$

$$\boxed{a_1 = 347.77 \text{ m/s}}$$

$$M_1 = \frac{80}{347.77} \Rightarrow M_1 = 0.23$$

$$\frac{P_1}{P} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{250\times10^5}{700\times10^5} = \left[\frac{1+\frac{1.4-1}{2}(0.23)^2}{1+\frac{1.4-1}{2}(M_2^2)}\right]^{\frac{1.4}{1.4-1}}$$

$$0.357 = \left(\frac{1.01058}{1+0.2M_2^2}\right)^{3.5}$$

$$0.357 = \frac{(1.01058)^{3.5}}{(1+0.2M_2^2)^{3.5}}$$

$$(1+0.2M_2^2)^{3.5} = 2.906.$$

$$1+0.2M_2^2 = (2.906)^{1/3.5} = 1.357$$

$$0.2M_2^2 = 1.357-1$$

$$M_2^2 = 1.785$$

$$\boxed{M_2 = 1.34}$$

The flow is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \boxed{T_2 = 224.45\,K}$$

$$a_2 = \sqrt{\gamma R T_2}$$

$$= \sqrt{1.4 \times 287 \times 224.45}$$

$$\boxed{a_2 = 300.31\,m/s}$$

$$M_2 = \frac{C_2}{a_2}$$

$$C_2 = M_2 \times a_2$$

$$= 1.34 \times 300.31$$

$$\boxed{C_2 = 402.42\,m/s}$$

Nozzles:

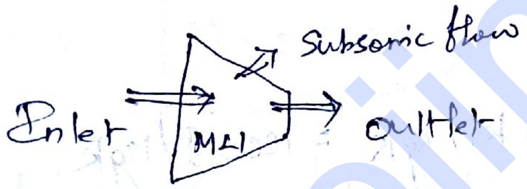
Nozzle is a duct of varying cross sectional area in which the velocity of fluid increasing with the corresponding pressure drop.

Types of nozzles:

(i) Convergent nozzles:

In this nozzle the C/S area decreases from the inlet to the outlet.

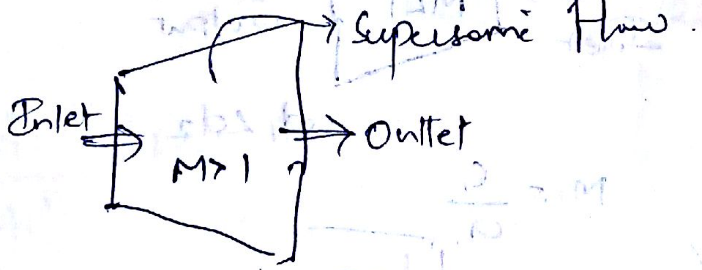
ie, $M<1$ & $dA = -ve$.



(ii) Divergent nozzle:

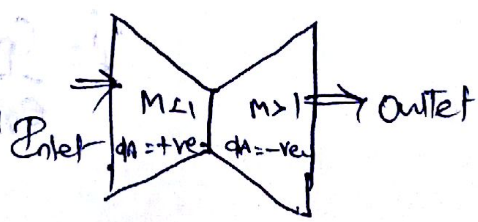
C/S area increases from the inlet to outlet.

ie, $M>1$, $dA = +ve$.



(iii) Convergent - Divergent nozzle (C-D) nozzle:

C/S first decreases from inlet to throat & then increases from throat to outlet.



3) A conical diffuser has entry & exit diameters are 15cm & 30cm. The pressure & velocity of air at entry are 0.69 bar, 340 K, and 180 m/s. Determine, i) the exit pressure ii) Exit velocity iii) the force exerted on the diffuser walls.

Assume isentropic flow $\gamma = 1.4$, $C_p = 1.00$ kJ/kgK

Given Data:

$d_1 = 15cm = 0.15m$

$d_2 = 30cm = 0.3m$

$P_1 = 0.69$ bar $= 0.69 \times 10^5$ N/m^2.

$T_1 = 340k$.

$C_1 = 180$ m/s

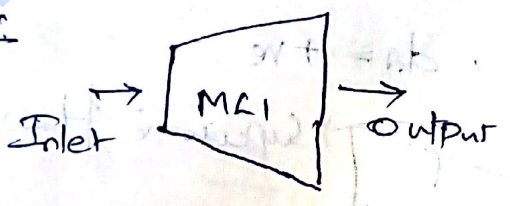
$\gamma = 1.4$

$C_p = 1.00$ kJ/kgK $= 1000$ J/kgK.

To find:

P_2, C_2, $T = F_2 - F$

Soln:



$d_1 < d_2$

$M_1 = \frac{C_1}{a_1}$

$a_1 = \sqrt{\gamma R T_1}$

$\gamma = \frac{C_p}{C_v}$, $C_v = 714.29$ J/kgk.

$$R = C_p - C_v$$

$$= 1000 - 714.29$$

$$\boxed{R = 285.71\ J/kg.K}$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 285.71 \times 340}$$

$$\boxed{a_1 = 368.78\ m/s}$$

$$M_1 = \frac{C_1}{a_1} = \frac{180}{368.78}$$

$$\boxed{M_1 = 0.49}$$

From gas table Page 29, Isentropic flow for $\gamma = 1.4$ & $M = 0.49$

M	T/T_0	P/P_0	A/A^*	F_1/F
0.49	0.954	0.848	1.360	1.126

$$\frac{A_1}{A_1^*} = 1.360$$

$$A_1^* = \frac{A_1}{1.360}$$

$$A_1 = \frac{\pi}{4} d_1^2 = 0.0166 m^2$$

$$A_1^* = 0.01302 m^2.$$

For isentropic flow the stagnation area remain const.

$S_0 = A_1^* = A_2^*$

$$A_2^* = 0.01302\ m^2.$$

At Exit, $A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.3)^2$

$$A_2 = 0.0707\ m^2$$

$$\frac{A_2}{A_2^*} = \frac{0.0707}{0.01302}$$

$$\boxed{\frac{A_2}{A_2^*} = 5.43}$$

From gas table for $\frac{A_2}{A_2^*} = 5.43 \simeq 5.299$ & $\gamma = 1.4$

(M<1 because divergent diffuser)

M	T/T₀	P/P₀	A/A*	F/F*
0.11	0.9976	0.992	5.299	4.915

$$\frac{P_2}{P_{02}} = 0.992 \Rightarrow P_2 = 80.717 \times 10^3\ N/m^2.$$

$$\frac{P_1}{P_{01}} = 0.848, \quad P_{01} = P_{02} = 0.8138 \times 10^5\ N/m^2.$$

$$\frac{T_1}{T_{01}} = 0.954$$

$$T_{01} = \frac{T_1}{0.954} \Rightarrow \boxed{T_{01} = 356.39\ K}$$

$$T_{01} = T_{02} = T_0 = 356.39\ K.$$

$$\frac{T_2}{T_{02}} = 0.9976$$

$$T_2 = 0.9976 \times T_{02}$$

$$\boxed{T_2 = 355.54\ k}$$

$$M_2 = \frac{C_2}{a_2} \Rightarrow a_2 = \sqrt{\gamma R T_2}$$

$$\boxed{a_2 = 377.11\ m/s}$$

$$M_2 = \frac{C_2}{a_2} \Rightarrow C_2 = M_2 \times a_2$$

$$= 0.11 \times 377.11$$

$$\boxed{C_2 = 41.48\ m/s}$$

Table (1) $\tau = F_2 - F_1$

$$\frac{F_1}{F_1^*} = 1.126 \Rightarrow F_1 = 1.126\ F_1^*$$

Table (2)

$$\frac{F_2}{F_2^*} = 4.215$$

$$F_2 = 4.215\ F_2^*. \qquad (\because F_2^* = F_1^*)$$

$$\tau = 3.089 F_1^*$$

$$F_1^* = p_1^* A_1^* (1+\gamma)$$

$$p^* = 42.962 \times 10^3\ N/m^2 //.$$

$F_1^* = P_1^* A_1^* (1+\gamma)$

$= 42.962 \times 10^3 \times 0.0132 \, (1+1.4)$

$F_1^* = 1361.05 \, N.$

$\tau = 3.089 \times F_1$

$= 3.089 \times 1361.05$

$\boxed{\tau = 4204.27 \, N}$

$\therefore$ Force exerted on the diffuser wall is $4204.27 \, N$.

Unit - II

Flow Through Ducts

Flow through Constant Area ducts with heat transfer (Rayleigh flow)

A frictionless flow in a constant area duct with heat transfer is known as "Rayleigh flow"

(E.g.): Heat Exchangers
Combustion Chambers
Inter Coolers
Regenerators

Assumptions:

(i) The flow takes place in constant area duct.

(ii) The gas is a Perfect gas

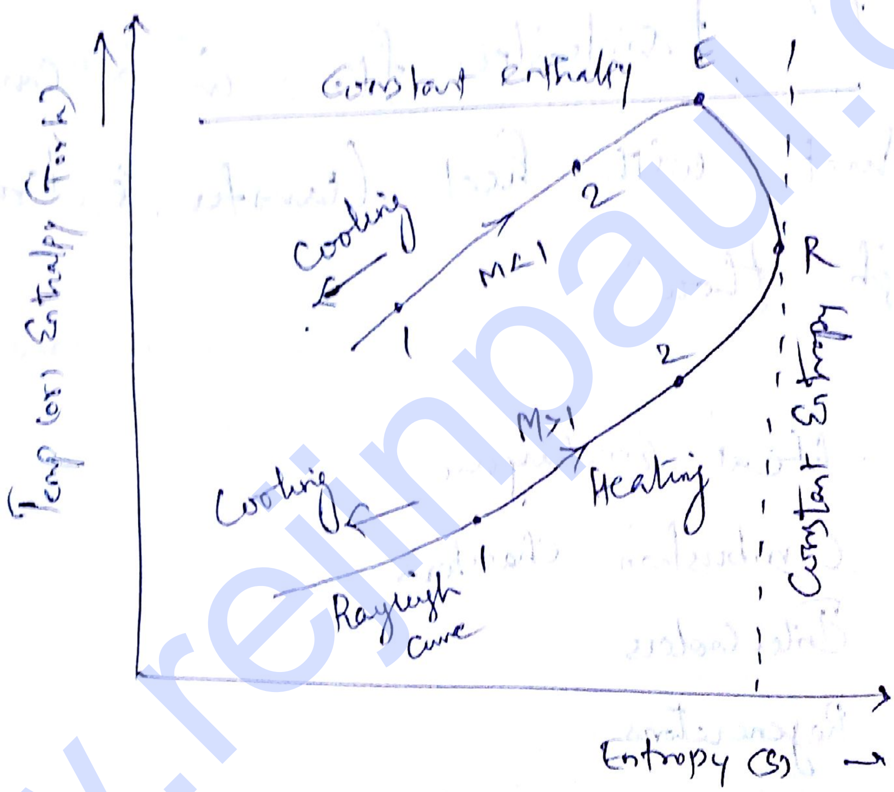
(iii) One dimensional, steady flow

(iv) There is no friction

(v) Absence of body forces (No work transfer)

Rayleigh line (or) Rayleigh Curve:

The frictionless flow in a constant area duct with heat transfer is described by a curve known as "Rayleigh line (or) Rayleigh Curve".



Rayleigh flow relations

(i) Impulse function $\frac{F_2}{F_1} = 1$

(ii) Stagnation Pressure

$$\frac{P_{o2}}{P_{o1}} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} \times \frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1}}$$

(iii) Static temperature

$$\frac{T_2}{T_1} = \frac{M_2^2}{M_1^2}\left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2$$

(iv) Stagnation temperature

$$\frac{T_{02}}{T_{01}} = \frac{M_2^2}{M_1^2}\left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \times \frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}$$

(v) Density ratio :

$$\frac{\rho_2}{\rho_1} = \frac{M_1^2}{M_2^2} \times \frac{1+\gamma M_2^2}{1+\gamma M_1^2}$$

(vi) Change of Entropy

$$\frac{S_2 - S_1}{R} = \ln \frac{M_2}{M_1}\left[\frac{M_2}{M_1}\left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

(vii) Heat transfer.

$$\frac{Q}{C_p T_1} = \frac{M_2^2 - M_1^2}{2M_1^2(1+\gamma M_2^2)^2}\left[2(1-\gamma M_1^2 M_2^2) + (\gamma-1)(M_2^2+M_1^2)\right]$$

—

1) Air enters a constant area duct at $M_1 = 3$, $P_1 = 1$ atm, and $T_1 = 300$ K. Inside the duct the heat added per unit mass is $q = 3 \times 10^5$ J/kg. Calculate the flow properties M_2, P_2, T_2, ρ_2, T_{02} and P_{02} at the exit.

Given: Enter Mach number $M_1 = 3$

$P_1 = 1$ atm $= 1 \times 10^5$ N/m^2

$T_1 = 300$ K.

$q = 3 \times 10^5$ J/kg

$m = 1$ kg (unit mass)

Assume $\gamma = 1.4$

$R = 287.5$ J/kg K

$C_p = 1005$ J/kg K

for air

To find:

M_2, P_{02}

P_2,

T_2,

ρ_2,

T_{02},

Soln:

Constant area duct with heat transfer and without friction. (Rayleigh flow Problem)

From gas table page 37,

Refer isentropic flow for $\gamma = 1.4$, & $M_1 = 3$

M	T/T_0	P/P_0
3	0.357	0.0272

$$\frac{T_1}{T_{01}} = 0.357$$

$$T_{01} = \frac{T_1}{0.357}$$

$$T_{01} = \frac{300}{0.357}$$

$$\boxed{T_{01} = 840.34\,K}$$

$$\frac{P_1}{P_{01}} = 0.0272$$

$$P_{01} = \frac{P_1}{0.0272}$$

$$= \frac{1 \times 10^5}{0.0272}$$

$$\boxed{P_{01} = 3.6765 \times 10^6\ N/m^2}$$

From gas table Page 115, Refer Rayleigh flow for $\gamma = 1.4$ & $M_1 = 3$

M	p/p^*	P_0/P_0^*	T/T_0^*	T_0/T_0^*	ρ^*/ρ
3	0.176	3.424	0.281	0.654	1.588

$$\frac{P_1}{P_1^*} = 0.176$$

$$P_1^* = \frac{P_1}{0.176} = \frac{1 \times 10^5}{0.176}$$

$$P_1^* = 5.68182 \times 10^5 \ N/m^2.$$

$\because P_1^* = P_2^*$

$$P_2^* = 5.68182 \times 10^5 \ N/m^2$$

$$\frac{P_{01}}{P_{01}^*} = 3.424$$

$$P_{01}^* = \frac{P_{01}}{3.424} = \frac{3.6765 \times 10^6}{3.424}$$

$$P_{01}^* = P_{02}^* = 1.0737 \times 10^6 \ N/m^2$$

$\because P_{01}^* = P_{02}^*$

$$\frac{T_1}{T_1^*} = 0.281$$

$$T_1^* = \frac{T_1}{0.281} = \frac{300}{0.281}$$

$T_1^* = T_2^*$

$$T_1^* = T_2^* = 1067.62 \ K$$

$$\frac{T_{01}}{T_{01}^*} = 0.654$$

$$T_{01}^* = \frac{T_1}{0.654} = \frac{840.34}{0.654}$$

$$\boxed{T_{02}^* = T_{01}^* = 1284.92\ K}$$

$$\frac{\rho_1^*}{\rho} = 1.588$$

$$\rho_1^* = 1.588 \times \rho = 1.588 \times \frac{P_1}{RT_1} \quad \left(\because \rho_1 = \frac{P_1}{RT_1}\right)$$

$$\rho_1^* = 1.588 \times \frac{1 \times 10^5}{287 \times 300}$$

$$\boxed{\rho_2^* = \rho_1^* = 1.844\ kg/m^3}$$

Heat transfer $(Q) = m\,C_p\,(T_{02} - T_{01})$

$$Q = 1 \times 1.005\,(T_{02} - 840.34)$$

$$3 \times 10^5 = 1005\,T_{02} - 844.5417 \times 10^3$$

$$\boxed{T_{02} = 1138.85\ K}$$

$$\frac{T_{02}}{T_{02}^*} = \frac{1138.85}{1284.92} = 0.886$$

from gas table Page 113. Refer Rayleigh flow for

$\gamma = 1.4$ & $\frac{T_{o2}}{T_{o2}^*} = 0.886 \simeq 0.884$

M	P/p^*	P_o/P_o^*	T/T^*	T_o/T_o^*	ρ^*/ρ
1.6	0.524	1.176	0.702	0.884	1.340

$$\frac{P_2}{P_2^*} = 0.524$$

$$P_2 = 0.524 \times P_2^* = 0.524 \times 5.68 \times 10^5$$

$$P_2 = 2.9773 \times 10^5 \ N/m^2$$

$$\frac{P_{o2}}{P_{o2}^*} = 1.176$$

$$P_{o2} = 1.176 \times P_{o2}^* = 1.176 \times 1.0737 \times 10^6$$

$$\boxed{P_{o2} = 1.2627 \times 10^6 \ N/m^2}$$

$$\frac{T_2}{T_2^*} = 0.702$$

$$T_2 = 0.702 \times T_2^*$$

$$= 0.702 \times 1067.62$$

$$\boxed{T_2 = 749.47 K}$$

$$\frac{\rho_2^*}{\rho_2} = 1.340$$

$$\rho_2 = \frac{\rho_2^*}{1.340} = \frac{1.844}{1.340}$$

$$\boxed{\rho_2 = 1.376 \text{ kg/m}^3}$$

2) The stagnation temperature of air is raised from 85°C to 376°C in a heat exchanger. If the inlet mach number is 0.4. Determine the final mach number & Percentage drop in Pressure.

Given:

$T_{01} = 85°C + 273 = 358\text{ K}$.

$T_{02} = 376° + 273 = 649\text{ K}$.

Inlet Mach number $M_1 = 0.4$

To find: $M_2 = ?$ (Find Mach number)

Precentage drop in pressure = ?

Soln:

Assume $\gamma = 1.4$ for air

$R = 287\ J/kg.K$

From gas table page 111, Refer Rayleigh flow

for $\gamma = 1.4$ & $M_1 = 0.4$

M	P/P^*	P_0/P_0^*	T/T^*	T_0/T_0^*
0.4	1.961	1.157	0.615	0.529

$$\frac{T_{01}}{T_{01}^*} = 0.529$$

$$T_{01}^* = \frac{T_{01}}{0.529} = \frac{358}{0.529}$$

$$T_{01}^* = T_{02}^* = 676.75\ K.$$

$$\frac{T_{02}}{T_{02}^*} = \frac{649}{676.75}$$

$$\frac{T_{02}}{T_{02}^*} = 0.958 \approx 0.955$$

From gas table page 112, Refer Rayleigh flow

for $\gamma = 1.4$ $\frac{T_{02}}{T_{02}^*} = 0.958 \approx 0.955$

M	P/P^*	P_0/P_0^*	T/T^*	T_0/T_0^*
0.78	1.296	1.023	1.022	0.955

$$\boxed{M_2 = 0.78}$$

$$\frac{P_2}{P_2^*} = 1.296.$$

$$\text{Percentage drop in Pressure} = \frac{\frac{P_1}{P_1^*} - \frac{P_2}{P_2^*}}{\frac{P_1}{P_1^*}} \times 100$$

$$= \frac{1.961 - 1.296}{1.961} \times 100.$$

$$= 33.91\%.$$

Flow through Constant area duct with friction & without heat transfer.

(Fanno flow)

(Ex)

(i) Air conditioning system

(ii) Industrial plants

(iii) Flow machineries

(iv) Flow process occuring in gas ducts of aircraft engines.

Assumptions:

(i) Flow in a Constant area duct,

(ii) The gas is Perfect gas

(iii) One dimensional steady flow

(iv) Absence of heat & work Transfer

(v) Flow is adiabatic

Fannoline (OR) fanno curve:

Flow in a Constant area duct with friction and without heat transfer is described by a curve is called "fanno line" or "fanno curve".

1) Air at an inlet temperature of 60°c flows with Subsonic velocity through an insulated Pipe having inside diameter of 50mm and length of 5m. The Pressure at the exit of the Pipe is 101 kpa and the flow is choked at the end of the Pipe. If the friction factor 4f = 0.05, determine the inlet mach number, the mass flow rate and the exit temperature.

Given:

$T_1 = 60°c + 273 = 333k$

$D = 50mm = 0.05m$

$L = 5m$

$P_2 = 101 \text{ kpa} = 101 \times 10^3 \text{ N/m}^2$.

$4f = 0.005$.

Flow is chocket at the end. i.e, Mach number value is one.

So, $M_2 = 1$.

To find:

$M_1 = ?$

$m = ?$

$T_2 = ?$

Soln: With friction & without heat transfer, so it is Fanno flow problem.

Assume $\gamma = 1.4$ } for air.
$R = 287\ J/kg.K$

Page 89, Refer fanno flow $\gamma = 1.4$, $M_2 = 1$.

M	P/p^*	T/T^*	P_0/P_0^*	$\frac{4\bar{f}L_{max}}{D}$
1	1	1	1	0

$$\frac{4fL}{D} = \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_1} - \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2}$$

$$\frac{0.005 \times 5}{0.05} = \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_1} - 0.$$

$$\left[\frac{4\bar{f}L_{max}}{D}\right]_{M_1} = 0.5$$

$$\frac{P_2}{P_2^*} = 1$$

$$P_2 = P_2^*$$

$$P_1^* = P_2 = P_2^* = 101 \times 10^3\ N/m^2 \quad \because P_1^* = P_2^*$$

$$\frac{T_2}{T_2^*} = 1, \quad T_2 = T_2^*$$

From gas table Page 81, Refer Fanno flow

$\gamma = 1.4$ and $\left[\frac{4\bar{f} L_{max}}{D}\right] = 0.5 = 0.491$

M	P/P^*	T/T^*	P_0/P_0^*	P_0/P_0^*
0.6	1.763	1.119	1.188	0.491

$$\frac{P_1}{P_1^*} = 1.763$$

$$P_1 = 1.763 \times P_1^* = 1.763 \times 101 \times 10^3.$$

$$\Rightarrow \quad P_1 = 1.78 \times 10^5 \ N/m^2$$

$$\frac{T_1}{T_1^*} = 1.119$$

$$T_1 = 1.119 \times T_1^*$$

$$T_1^* = \frac{T_1}{1.119} = \frac{333}{1.119}$$

$$T_1^* \boxed{= 297.59 \ K.} \qquad T_1^* = T_2$$

(or)

$$T_2 = 297.89 \ K$$

$$m = \rho A C = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$$

$$A_1 = \frac{\pi}{4} (D_1)^2$$

$$= \frac{\pi}{4} (0.05)^2$$

$$A_1 = 1.9635 \times 10^{-3} \, m^2.$$

$$m = \rho_1 A_1 C_1$$

$$M_1 = \frac{C_1}{a_1}$$

$$a_1 = \sqrt{\gamma R T_1}$$

$$= \sqrt{1.4 \times 287 \times 333}$$

$$\boxed{a_1 = 365.79 \, m/s}$$

$$M_1 = \frac{C_1}{a_1}$$

$$C_1 = M_1 \times a_1$$

$$= 0.6 \times 365.79$$

$$\boxed{C_1 = 219.47 \, m/s}$$

$$m = \frac{P_1}{RT_1} \times D_1 \times C_1 = \frac{1.78 \times 10^5}{287 \times 333} \times 1.9653 \times 10^{-3} \times 219.47$$

$$\boxed{m = 0.803 \, kg/s}$$

Variation of flow Properties

Flow Properties like Pressure (P), Temperature (T) density (ρ) and fluid velocity (C) at $M = M^* = 1$ are used as reference values for non-dimensionalizing various properties at any section of the duct.

1. Temperature :

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2}$$

2. velocity

$$\frac{C_2}{C_1} = \frac{M_2}{M_1} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{1/2}$$

3. Density,

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right]$$

4. pressure

$$\frac{P_2}{P_1} = \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{1/2}$$

Stagnation Pressure:

$$\frac{P_{o_2}}{P_{o_1}} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Impulse function:

$$\frac{F_2}{F_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}\left[\frac{1+\gamma M_2^2}{1+\gamma M_1^2}\right]$$

Change of entropy:

$$\frac{S_2-S_1}{R} = \ln\frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

—

Unit - III

Normal and oblique shocks

Shock wave:

A shock wave is a special kind of wave refered as a steep finite pressure wave. The changes in the flow properties across such a wave are abrupt.

Types of shock waves:

(1) According to angular position

(i) Normal shock wave

(ii) Oblique shock wave

(iii) Curved shock wave.

(2) According to motion

(i) Stationary shock wave

(ii) Moving shock wave

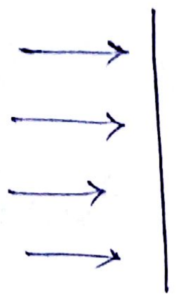
(3) According to the strength

(i) weak waves

(ii) Strong waves.

Normal shock wave:

- When the shock wave is at right angle to the flow.
- It is an one dimensional flow.



(normal shock wave)

Oblique shock wave:

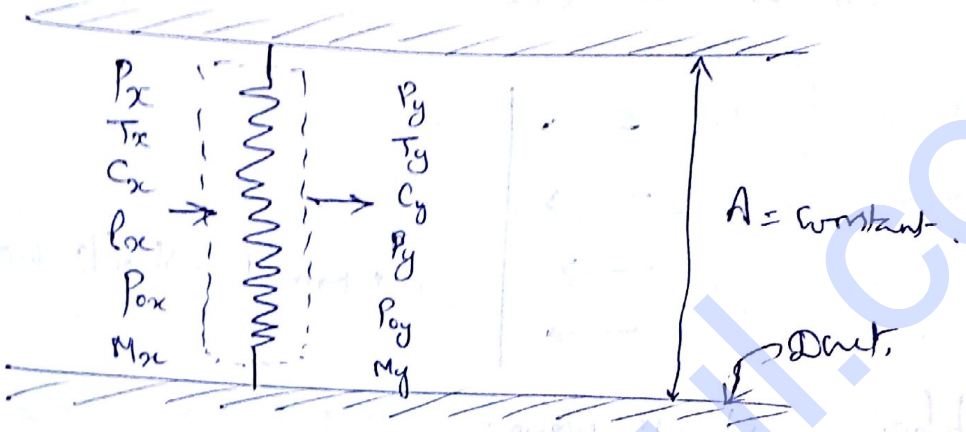
- When the shock wave is inclined at an angle to the flow.
- Two dimensional flow.

Governing equation:

Assumptions:

(i) Flow in a constant area duct.
(ii) The flow is a frictionless flow
(iii) The gas as a perfect gas
(iv) The shock wave is perpendicular to the flow stream lines.

Let us consider a control volume of thickness "dx" in the flow direction for the analysis of normal shock wave as shown in fig.



Continuity equation for the flow through control volume is given by,

$$\dot{m} = \rho_x A_x C_x = \rho_y A_y C_y.$$

For constant area duct $A_x = A_y = A = \text{Constant}$

$$\therefore \frac{\dot{m}}{A} = \rho_x C_x = \rho_y C_y.$$

For adiabatic flow with no external workdone, the steady flow energy equation gives,

$$h_{ox} = h_{oy} = h_o = \text{constant}$$

$$h_x + \frac{1}{2} C_x^2 = h_y + \frac{1}{2} C_y^2$$

Momentum equation gives.

$$(P_x - P_y)A = \dot{m}\,(C_y - C_x).$$

$$P_x - P_y = \frac{\dot{m}}{A}\,(C_y - C_x).$$

$$\therefore \quad P_x - P_y = \rho_x C_x (C_y - C_x) \qquad (\because \rho_x C_x = \rho_y C_y)$$

$$P_x + \rho_x C_x^2 = P_y + \rho_y C_y^2.$$

From the definition of impulse function:

$$F_x = F_y = \text{Constant}$$

The equation of state gives.

$$h = f(S, P)$$

$$S = f(P, \rho)$$

Variation of flow parameters across the normal & oblique shocks

Static pressure across the shock.

Force acting before the normal shock = Force acting after the normal shock

$$F_x = F_y.$$

$$\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)$$

Temperature ratio across the shock:

WKT

Stagnation temperature - Mach number relation.

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2.$$

$$\frac{T_y}{T_x} = \frac{\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]\left[1 + \frac{\gamma-1}{2} M_x^2\right]}{\frac{M_x^2}{2(\gamma-1)} \times (\gamma+1)^2}$$

Density across the shock:

WKT Density $\rho = \frac{p}{RT}$

$$\frac{\rho_y}{\rho_x} = \frac{p_y}{p_x} \times \frac{T_x}{T_y}$$

$$\frac{\rho_y}{\rho_x} = \frac{\frac{p_y}{p_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1}{\frac{\gamma+1}{\gamma-1} - \frac{p_y}{p_x}}$$

Stagnation Pressure ratio across the shock:

A shock wave is an irreversibility across which there is a stagnation pressure loss and increase in entropy.

$$\frac{P_{oy}}{P_{ox}} = \left[\frac{\frac{M_x^2}{2}(\gamma+1)}{1+\frac{\gamma-1}{2}M_x^2}\right]^{\gamma/\gamma-1} \times \left[\frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}\right]^{-\frac{1}{\gamma-1}}$$

Change in entropy across the shock:

WKT

Change in entropy $\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

$$\therefore \frac{\Delta S}{R} = -\frac{\gamma}{\gamma-1} \ln\left[\frac{\frac{\gamma+1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_x^2}\right] + \frac{1}{\gamma-1}\ln\left[\frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}\right]$$

Problems :

① Air flows adiabatically in a pipe. A normal shock wave is formed. The pressure & temperature of air before the shock are 150 kN/m² & 25°C. The pressure just after the normal shock is 350 kN/m². Calculate i) Mach number before shock ii) Mach number, static temp & velocity of air after the shock wave iii) Increase in density of air. iv) Loss of stagnation pressure of air (v) Change in entropy.

Given Data :

Upstream Pressure $P_x = 150 \text{ kN/m}^2 = 150 \times 10^3 \text{ N/m}^2$.

Upstream Temperature $T_x = 25 + 273 = 298\text{ K}$.

Downstream Pressure $P_y = 350 \text{ kN/m}^2 = 350 \times 10^3 \text{ N/m}^2$.

To find :

$M_x = ?$ $\quad \rho_y - \rho_x = ?$

$M_y = ?$ $\quad \Delta P_0 = ?$

$T_y = ?$

$C_y = ?$ $\quad \Delta s = ?$

Soln:

This Problem deals with normal shock.

Assume $\gamma = 1.4$
$R = 287\ J/kg.K$ } For Air

$$\frac{P_y}{P_x} = \frac{350 \times \cancel{10^3}}{150 \times \cancel{10^3}}$$
$$= 2.333.$$

From Gas table Page 57, Refer normal shock $\gamma = 1.4$

$$\frac{P_y}{P_x} = 2.333 \approx 2.320.$$

M_x	M_y	$\frac{P_y}{P_x}$	$\frac{T_y}{T_x}$	$\frac{P_{oy}}{P_{ox}}$	$\frac{P_{oy}}{P_x}$
1.46	0.716	2.320	1.294	0.942	3.265

$M_x = 1.46.$

$M_y = 0.716.$

$$\frac{T_y}{T_x} = 1.294$$

$T_y = 1.294 \times T_x = 1.294 \times 298$

$T_y = 385.61\ K.$

$$\frac{P_{oy}}{P_{ox}} = 0.942$$

$$\frac{P_{oy}}{P_x} = 3.265.$$

$P_{oy} = 3.265 \times P_x$

$= 3.265 \times 150 \times 10^3$

$P_{oy} = 4.8975 \times 10^5\ N/m^2.$

$$\frac{P_{oy}}{P_{ox}} = 0.942.$$

$$P_{ox} = \frac{P_{oy}}{0.942} = \frac{4.8975 \times 10^5}{0.942}$$

$$P_{ox} = 5.199 \times 10^5 \ N/m^2.$$

$$\Delta P_o = P_{ox} - P_{oy}$$

$$= 5.199 \times 10^5 - 4.8975 \times 10^5$$

$$\Delta P_o = 30.154 \times 10^5 \ N/m^2.$$

$$M_y = \frac{C_y}{a_y}$$

$$a_y = \sqrt{\gamma R T_y} = \sqrt{1.4 \times 287 \times 386.61}$$

$$a_y = 393.62 \ m/s$$

$$C_y = M_y \times a_y$$

$$= 0.716 \times 393.62.$$

$$C_y = 281.83 \ m/s$$

$$\rho_x = \frac{P_x}{RT_x} = \frac{150 \times 10^3}{287 \times 298}$$

$$\rho_x = 1.7539 \ kg/m^3.$$

Increase in Entropy density,

$$= \rho_y - \rho_x$$

$$= 3.1626 - 1.7539$$

$$= 1.4087 \text{ kg/m}^3$$

Change in Entropy

$$\Delta s = R \ln \left(\frac{P_{ox}}{P_{oy}} \right)$$

$$= 287 \ln \left(\frac{5.199 \times 10^5}{4.8975 \times 10^5} \right)$$

$$\boxed{\Delta s = 17.146 \text{ J/kg.K}}$$

(P) The state of gas ($\gamma = 1.3$, $R = 469$ kJ/kgK) upstream of a normal shock wave is given by the following data. $M_x = 2.5$, $P_x = 2$ bar, $T_x = 275$ K. Calculate the mach number, pressure, temperature of the gas down stream of the shock.

Given:

$\gamma = 1.3$

$R = 469$ kJ/kg K

Upstream mach number $M_2 = 2.5 = M_x$

Upstream pressure $P_x = 2$ bar $= 2 \times 10^5$ N/m

Upstream temperature $T_x = 275$ K.

To find:

$M_y = ?$

$P_y = ?$

$T_y = ?$

Soln: This problem deals with normal shock.

Refer Page 49, normal shock for $\gamma = 1.3$, $M_2 = 2.5$

M_x	M_y	$\frac{P_y}{P_x}$	$\frac{T_y}{T_x}$	$\frac{P_{oy}}{P_{ox}}$	$\frac{P_{oy}}{P_x}$
2.5	0.493	6.935	1.869	0.460	8.098.

$M_y = 0.493$

$$\frac{P_y}{P_x} = 6.935$$

$P_y = 6.935 \times P_x$.

$$= 6.935 \times 2 \times 10^5$$

$$P_y = 1.387 \times 10^6 \ N/m^2$$

$$\frac{T_y}{T_x} = 1.869$$

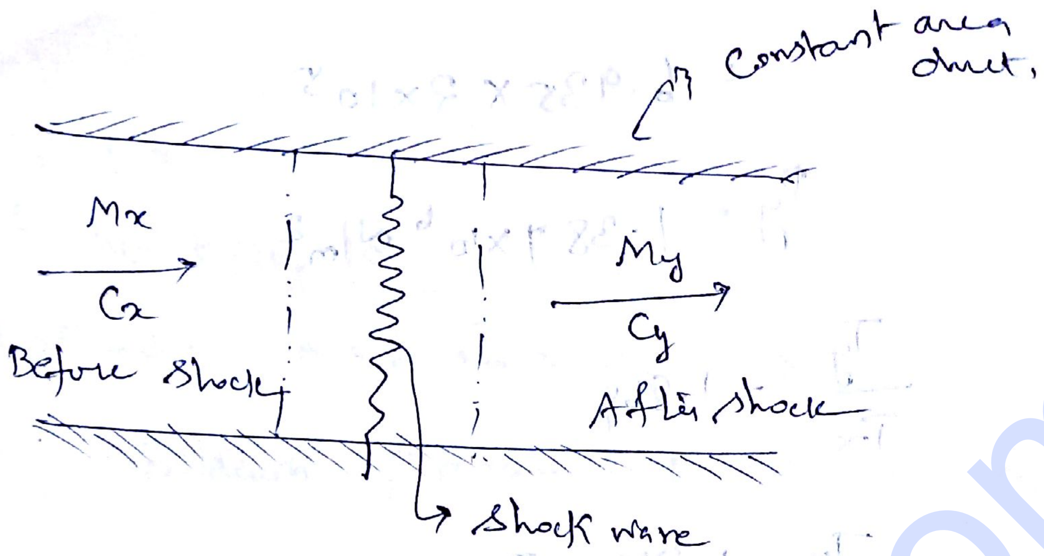
$$T_y = 1.869 \times T_x$$

$$T_y = 513.98 K.$$

Prandtl meyer Equation:

Prandtl meyer relation which is the basis of other equations for shock waves. It gives the relationship between the gas velocities before and after the normal shock and the critical velocity of sound.

Let us consider the control volume of before & after shock waves as shown in fig.



WKT

Stagnation enthalpy equation.

$$h_0 = \frac{a^2}{\gamma-1} + \frac{c^2}{2} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a^{*2}.$$

Applying this eqn. to the flow before & after shock wave, we get,

Before shock wave:

$$\frac{a_x^2}{\gamma-1} + \frac{1}{2} c_x^2 = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a^{*2}.$$

$$\frac{a_x^2}{\gamma-1} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a^{*2} - \frac{1}{2} c_x^2.$$

$$\frac{a_x^2}{\gamma-1} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a^{*2}.$$

$$a_x^2 = (\cancel{\gamma-1}) \times \frac{1}{2}\left(\frac{\gamma+1}{\cancel{\gamma-1}}\right) a^{*2} - \frac{1}{2} c_x^2(\gamma-1)$$

$$a_x^2 = \left(\frac{\gamma+1}{\gamma-1}\right) a^{*2} - \left(\frac{\gamma-1}{2}\right) C_x^2.$$

$\div$ "C_x"

$$\Rightarrow \frac{a_x^2}{C_x} = \left(\frac{\gamma+1}{2}\right) \frac{a^{*2}}{C_x} - \frac{\gamma-1}{2} C_x.$$

After shock wave.

$$\frac{a_y^2}{\gamma-1} + \frac{1}{2} C_y^2 = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a^{*2}.$$

$$\frac{a_y^2}{\gamma-1} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a^{*2} - \frac{1}{2} C_y^2.$$

$$a_y^2 = (\cancel{\gamma-1}) \cdot \frac{1}{2}\left(\frac{\gamma+1}{\cancel{\gamma-1}}\right) a^{*2} - \frac{1}{2} C_y^2 (\gamma-1)$$

$$a_y^2 = \left(\frac{\gamma+1}{2}\right) a^{*2} - \frac{\gamma-1}{2} C_y^2.$$

$\div$ by "C_y"

$$\Rightarrow \frac{a_y^2}{C_y} = \left(\frac{\gamma+1}{2}\right) \frac{a^{*2}}{C_y} - \frac{\gamma-1}{2} \cdot C_y.$$

From momentum equation,

$$(P_x - P_y) A = m (C_y - C_x)$$

$$P_x - P_y = \frac{m}{A} (C_y - C_x)$$

$$\frac{P_x - P_y}{\left(\frac{m}{A}\right)} = C_y - C_x .$$

$$\frac{P_x}{\left(\frac{m}{A}\right)} - \frac{P_y}{\left(\frac{m}{A}\right)} = C_y - C_x .$$

WKT mass flow rate, $m = \rho A C$.

$$= \rho_x A_x C_x = \rho_y A_y C_y .$$

$$\frac{m}{A} = \rho_x C_x = \rho_y C_y . \quad (\because A = \text{const})$$

$$\Rightarrow \frac{P_x}{\rho_x C_x} - \frac{P_y}{\rho_y C_y} = C_y - C_x .$$

Multiplying "γ" on both sides.

$$\frac{\gamma P_x}{\rho_x C_x} - \frac{\gamma P_y}{\rho_y C_y} = \gamma (C_y - C_x).$$

From gas equation $PV = mRT$.

$$PV = RT \quad \therefore (m = 1\,kg)$$

$$\rho = \frac{1}{v}$$

$$\frac{P}{\rho} = RT \qquad a = \sqrt{\gamma RT}$$

$$\frac{P}{\rho} = \frac{\gamma RT}{\gamma} \qquad a^2 = \gamma RT$$

$$\frac{P}{\rho} = \frac{a^2}{\gamma}$$

$$\frac{\gamma P}{\rho} = a^2$$

$$\frac{\gamma P_x}{\rho_x} = a_x^2.$$

Similarly,

$$\frac{\gamma P_y}{\rho_y} = a_y^2.$$

Sub, a_x^2 & a_y^2,

$$\frac{a_x^2}{C_x} - \frac{a_y^2}{C_y} = \gamma(C_y - C_x).$$

$$\left(\frac{\gamma+1}{2}\right) a^{*2} \cdot \left(\frac{C_y - C_x}{C_x \cdot C_x}\right) + \left(\frac{\gamma-1}{2}\right)(C_y - C_x) = \gamma(C_y - C_x)$$

(X) by $\frac{C_x \cdot C_y}{C_y - C_x}$ on both sides

$$\left(\frac{\gamma+1}{2}\right) a^{*2} + \left(\frac{\gamma-1}{2}\right) \cdot C_x \cdot C_y = \gamma (C_x \cdot C_y).$$

(X) by 2 on both sides,

$$(\gamma+1) a^{*2} + (\gamma-1) C_x \cdot C_y = 2\gamma(C_x \cdot C_y)$$

$$(\gamma+1)\,a^{*2} = 2\gamma(C_x C_y) - [(\gamma-1)C_x C_y]$$

$$= 2\gamma C_x C_y - \gamma C_x C_y + C_x C_y$$

$$= \gamma C_x \cdot C_y + C_x C_y$$

$$(\gamma+1)\,a^{*2} = C_x C_y (\gamma+1)$$

$$a^{*2} = \frac{C_x C_y \cancel{(\gamma+1)}}{\cancel{(\gamma+1)}}$$

$$\boxed{a^{*2} = C_x C_y}$$

The above equation is known as Prandtl - meyer relation.

Unit - IV

Jet Propulsion

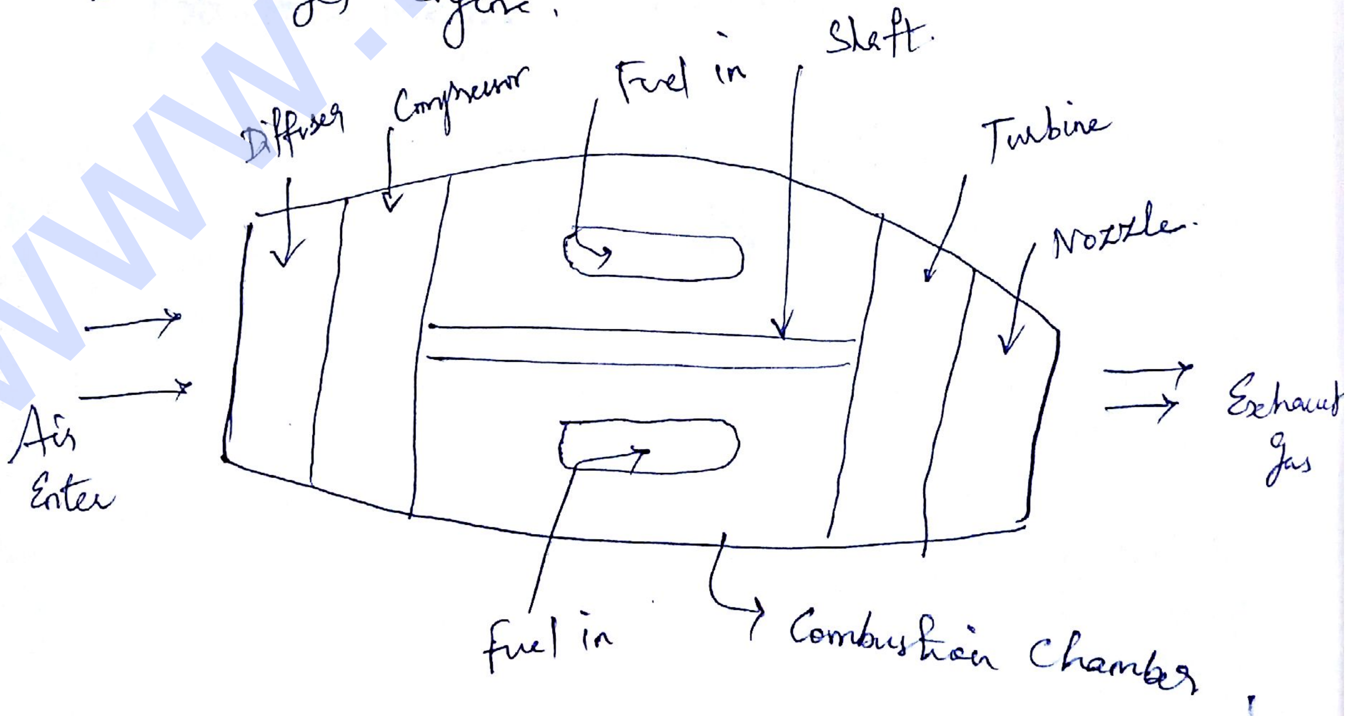
Jet Propulsion is the driving forward of a body by means of a jet of gas or fluid.

Principle of Jet Propulsion:

"It is obtained from "newton's third law of motion". It states that "for every action there is an equal and opposite reaction".

Turbojet Engine:

- The most widely used air-breathing engine is turbojet engine.



Working:

- The air from the atmosphere enters into the diffuser at the speed of vehicle. The diffuser is to reduce the velocity of incoming air and increases the static pressure. The air from the diffuser enters the compressor. It is further compressed to higher pressure. Rotary compressors are used in this engine. The high pressure air flows into the combustion chamber. In this combustion chamber, the fuel is injected by an injector & the air-fuel mixture is burnt. The heat is supplied at constant pressure.

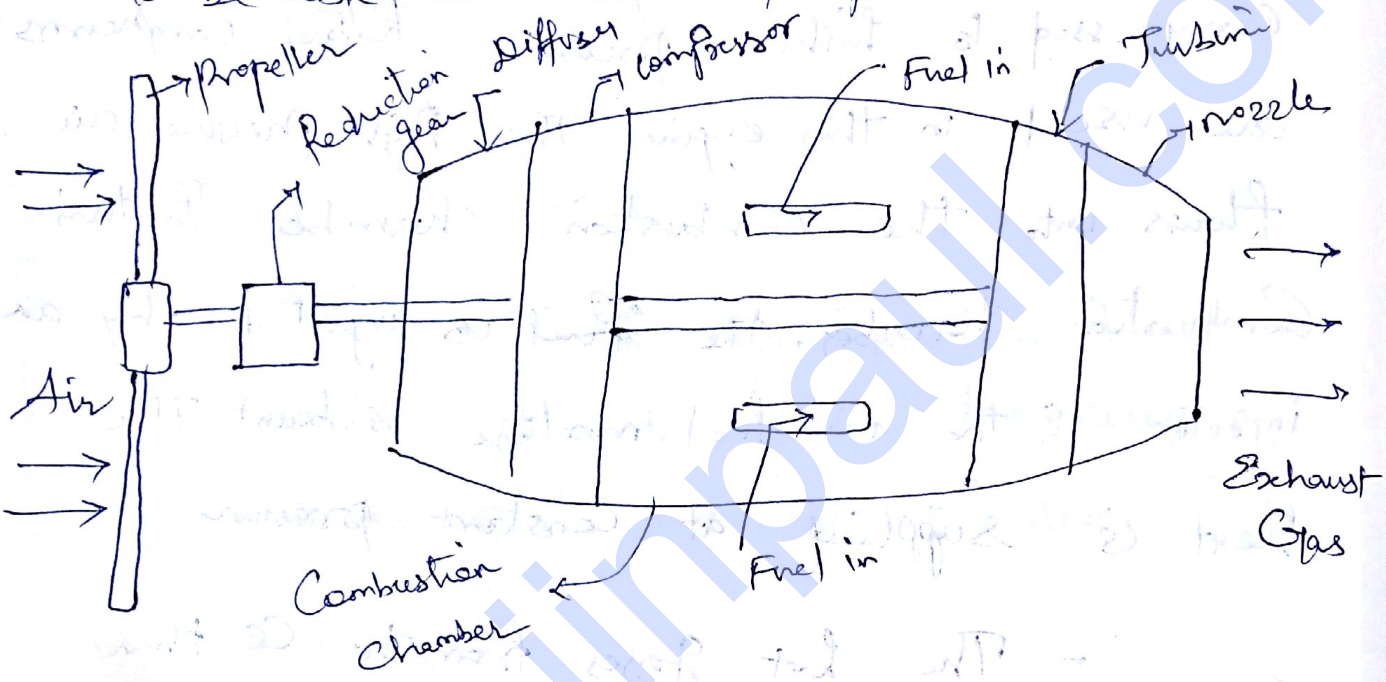
- The hot gases from the CC then expanded in the gas turbine to produce mechanical work. The work produced by the turbine is utilised to run the compressors & other auxiliaries in the turbojet engine. From the turbine the gas enters the nozzle.

- In the nozzle, pressure energy of the gas is converted into kinetic energy. The high velocity gas leaves from the nozzle, which propells the turbojet engine to move forward.

- The exhaust gases from the nozzle have a large quantity of oxygen which can support the combustion of fuel.

Turbo Prop Engine (or) Turbo Propeller Engine

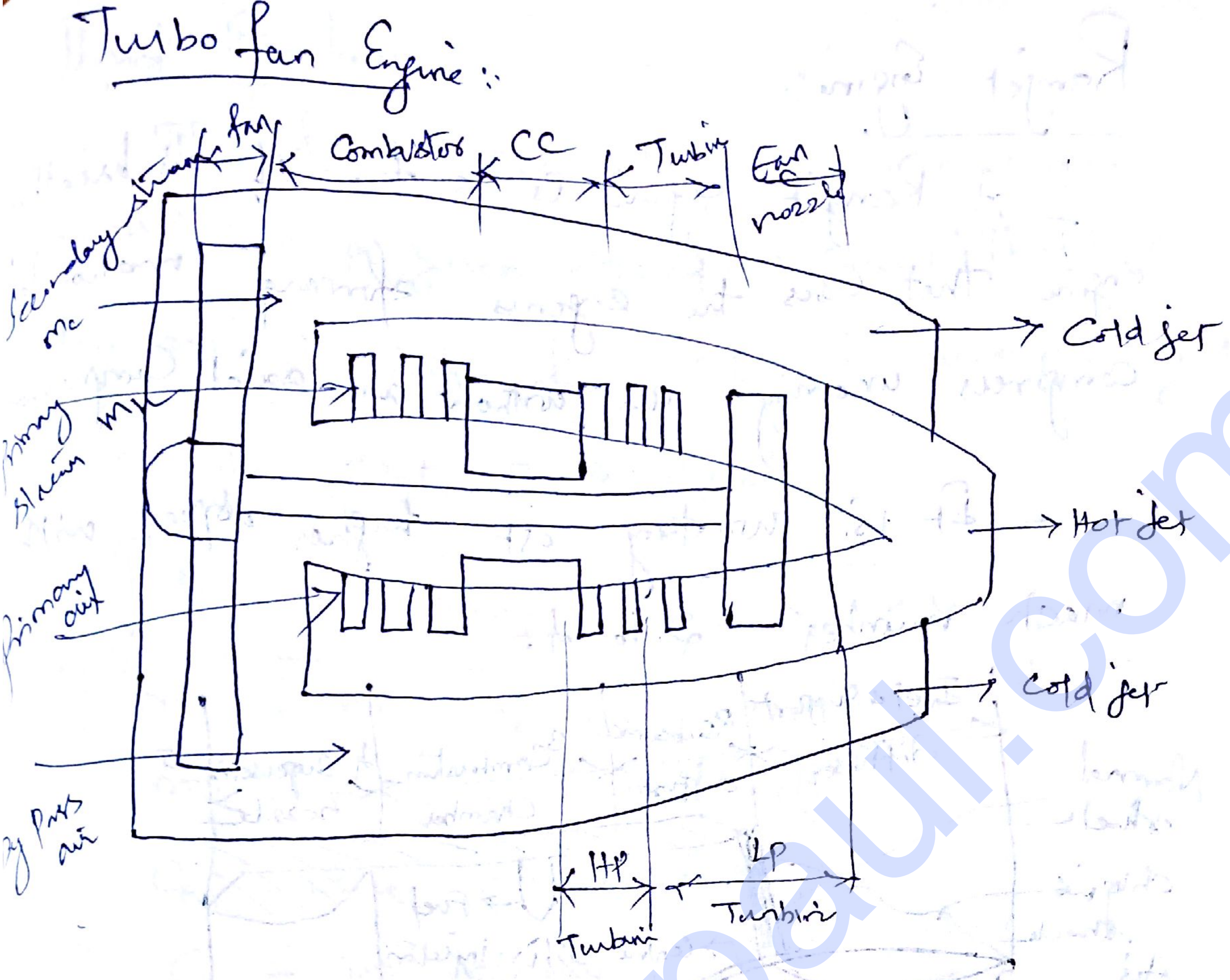
In turbo prop engines a conventional aircraft propeller is usually mounted in front of the jet engine. It is similar to turbojet engine. In this type, the turbine develops enough power to be able to drive the propeller & compressor.



Working:

The working principle is same as Turbojet Engine. In nozzle the pressure energy of the gas is converted into kinetic energy. (Very high velocity of gases are coming out). Due to this high velocity of gases, a reaction (or) thrust is produced in the opposite direction. (Sum of thrust produced by propeller and nozzle). This total thrust propels the air craft.

Turbo fan Engine ::



The ratio of the mass flow rates of cold air (m_c) and the hot air (m_h) is known as "by pass ratio"

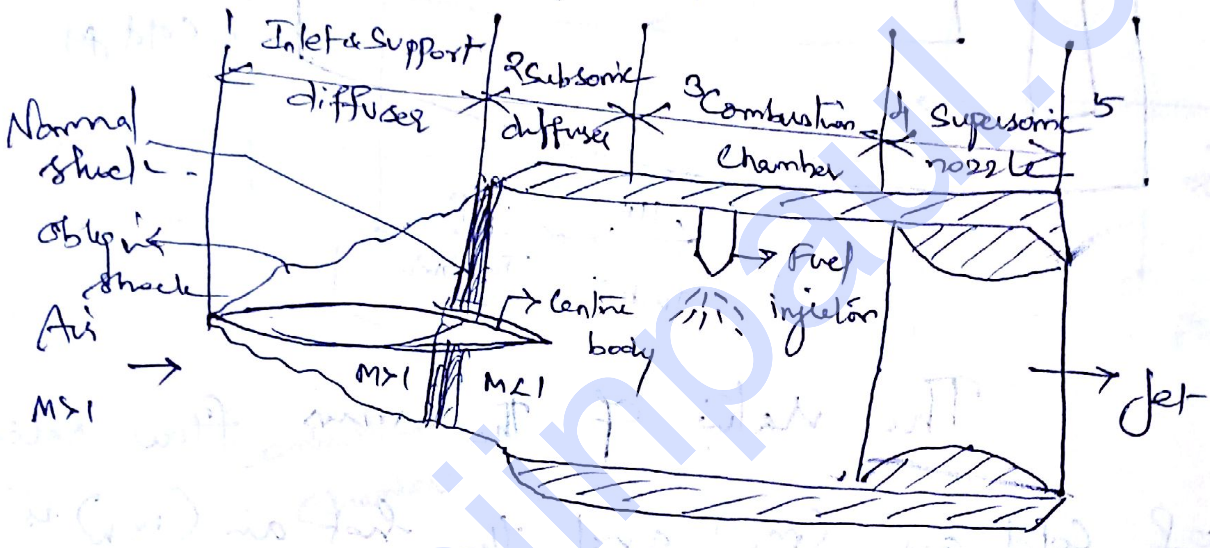
$$B = \frac{m_c}{m_h}$$

Adv:
i) Less noise is produced
ii) High efficiency.
iii) High take-off thrust.
iv) Thrust is higher than turbojet engine.

Ramjet Engine:

- A Ramjet engine is a form of air breathing engine that uses the engines forward motion to compress incoming air without an axial Compressor.

- It is working at higher speed with mach number - 2 to 4.



Adv:

(i) Construction is simple.

(ii) It can operates at supersonic mach number.

(iii) Cost is low (iv) Less maintenance.

(v) Light weight when compared with turbojet engines.

Disadv:

i) Thermal efficiency is low.

ii) Take off thrust is zero.

iii) It is not suitable for subsonic speeds.

Thrust Equation:

The force which propels the aircraft forward at a given speed is called "Thrust (or) drag or propulsive force. This is developed by the jet at the exit of the nozzle.

$$\boxed{F = (\dot{m}_a + \dot{m}_f) C_e - \dot{m}_a \times u.}$$

$\dot{m}_a$ - Mass of air in kg/s

$\dot{m}_f$ - mass of fuel in kg/s.

C_e - Exit velocity of gas

C_j - Jet velocity

u - Flight speed.

Thrust Power & propulsive efficiency.

Propulsive efficiency is defined as the ratio of Propulsive Power (or) Thrust Power to the Power o/p of the engine. It is denoted by η_p.

$$\eta_p = \frac{2u}{C_j + u}.$$

Thermal efficiency:

$$\eta_t = \frac{\frac{1}{2}\dot{m}(C_j^2 - u^2)}{\dot{m}_f \times C_v.}$$

Problems - Jet Propulsion:

1) A Turbojet propels an aircraft at a speed of 900 km/hr while taking 3000 kg of air per minute. The isentropic enthalpy drop in the nozzle is 200 kJ/kg and the nozzle efficiency is 90%. The air and fuel ratio is 85. and the combustion efficiency is 95%. The Calorific value of fuel is 42,000 kJ/kg. Calculate i) Propulsive power ii) Thrust power iii) Thermal efficiency iv) Propulsive efficiency.

Given:

Speed $u = 900 \text{ km/hr} = 250 \text{ m/s}$.

mass of air $\dot{m}_a = 3000 \text{ kg/m} = 50 \text{ kg/s}$.

Enthalpy drop $\Delta h = 200 \text{ kJ/kg} = 200 \times 10^3 \text{ J/kg}$.

$\eta_N = 90\% = 0.9$.

Air fuel ratio $\frac{\dot{m}_a}{\dot{m}_f} = 85$.

Combustion efficiency $\eta_B = 95\% = 0.95$.

Calorific value $C.V = 42000 \text{ kJ/kg} = 42000 \times 10^3 \text{ J/kg}$

To find:

Propulsive Power $= (P) = ?$

$\eta_{thermal} = ?$

$\eta_P = ?$

Soln: $P = F \times u$.

$F = \dot{m} C_j - \dot{m}_a u$.

$C_j = \sqrt{2 \times \eta_N \times \Delta h}$

$= \sqrt{2 \times 0.90 \times 200 \times 10^3}$

$\boxed{C_j = 600 \text{ m/s}}$

$\dot{m} = \dot{m}_a + \dot{m}_f$

$\frac{\dot{m}_a}{\dot{m}_f} = 85$

$\dot{m}_f = \frac{\dot{m}_a}{85} = \frac{50}{85} = 0.588 \text{ kg/s}$.

$\dot{m} = \dot{m}_a + \dot{m}_f$

$= 50 + 0.588$

$\dot{m} = 50.588 \text{ kg/s}$.

$F = \dot{m} C_j - \dot{m}_a u$.

$= (50.588 \times 600) - (50 \times 250)$

$F = 17852.8 \text{ N}$.

$P = F \times u = 17852.8 \times 250$

$P = 4.4632 \times 10^6$ Watts

$\eta_t = \frac{\frac{1}{2} \dot{m} (C_j^2 - u^2)}{\eta_B \times \dot{m}_f \times CV} = 0.3207$ $\boxed{\eta_t = 32.07\%}$

$\eta_p = \frac{2u}{C_j + u} \Rightarrow \boxed{\eta_p = 58.82\%}$

(2) A flight speed of a turbojet is 800 km/hr at 10,000 m altitude. The density of air at that altitude is 0.17 kg/m3. The drag for the plane is 6.8 kN. The propulsive efficiency of the jet is 60%. Calculate SFC. Air-fuel ratio and jet velocity. Assume the calorific value of fuel as 45000 kJ/kg. & the overall efficiency is 18%.

Given Data:

Flight speed $u = 800\ km/hr = 222.22\ m/s$.

Altitude $Z = 10{,}000\ m$.

$\rho = 0.17\ kg/m^3$.

Drag on thrust $F = 6.8\ kN = 6.8 \times 10^3\ N$.

$\eta_p = 60\% = 0.6$.

$CV = 45000 \times 10^3\ J/kg$.

Overall efficiency $\eta_o = 18\% = 0.18$.

To find:

SFC = ?

$\frac{\dot{m}_a}{\dot{m}_f} = ?$

$C_j = ?$

Soln: SFC (or) TSFC $= \dfrac{\dot{m}_f}{F}$

$F = \dot{m} C_j - \dot{m}_a u.$

$\eta_p = \dfrac{2u}{C_j + u}.$

$0.6 = \dfrac{2 \times 222.22}{C_j + 222.22}$

$\boxed{C_j = 518.52 \text{ m/s.}}$

$\eta_o = \dfrac{F \times u}{\dot{m}_f \times C_v}$

$0.18 = \dfrac{6.8 \times 10^3 \times 222.22}{\dot{m}_f \times 45000 \times 10^3}.$

$\boxed{\dot{m}_f = 0.1865 \text{ kg/s}}$

$F = \dot{m} C_j - \dot{m}_a u$

$(\dot{m} = \dot{m}_a + \dot{m}_f)$

$\Rightarrow \boxed{\dot{m}_a = 22.62 \text{ kg/s}}$

$$\frac{\dot{m}_a}{\dot{m}_f} = \frac{22.62}{0.1865}$$

$$\boxed{\frac{\dot{m}_a}{\dot{m}_f} = 121.3}$$

$$SFC = \frac{\dot{m}_f}{F} = \frac{0.1865}{6.8 \times 10^3}$$

$$SFC = 2.7426 \times 10^{-5} \text{ kg/sN}$$

$$\Rightarrow \boxed{SFC = 0.0987 \text{ kg/hr-N}}$$

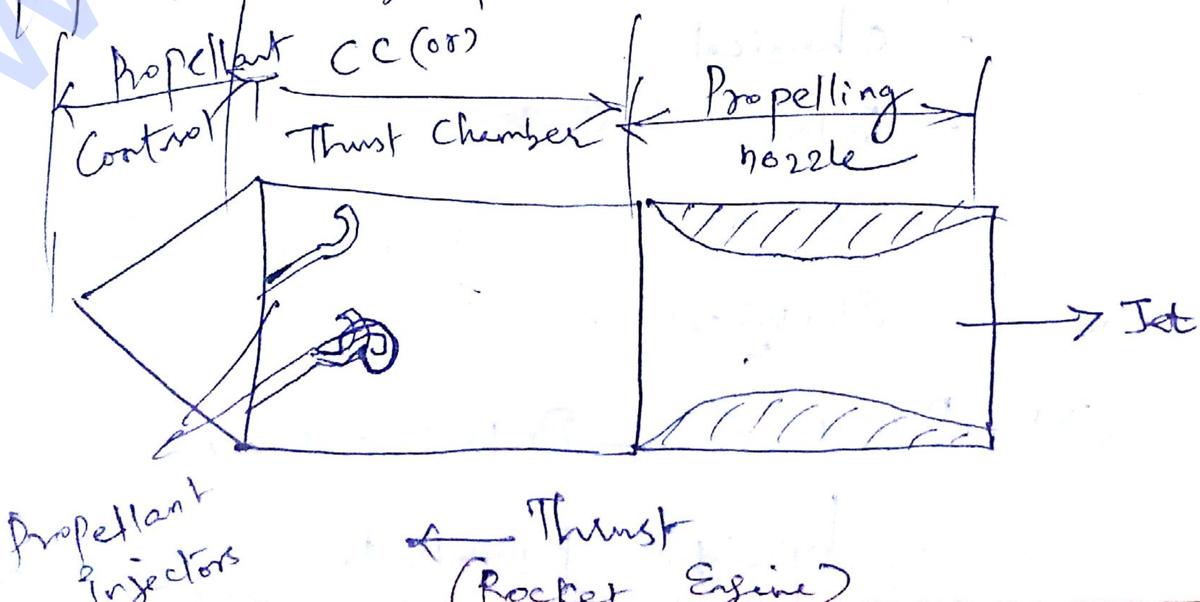
Unit - 5

Space Propulsion

- The basic working Principle of space (or) rocket Propulsion is similar to Jet Propulsion.

- In rocket Propulsion, the altitude of rocket engine is very high, so oxygen is not available in the atmosphere. So the oxygen is fitted in a tank in the rocket engine itself & used for combustion purposes.

- A rocket engine mainly consists of a container for propellants (fuel & oxidizer), combustion chamber (Thrust chamber) & a propulsion nozzle & the control and navigational equipments & payload as shown in fig.



Jet Propulsion	Space (or) Rocket Propulsion
(i) The oxygen required for combustion is taken from the atmosphere	The oxygen required for combustion takes place using it's own oxygen supply
(ii) It cannot operate in vacuum altitude.	It operate in vacuum.
(iii) It's performance depends on the altitude	not depends on the altitude
(iv) Flight speed is limited	not limited

Types of Rocket Engines

i) Based on source of energy employed

- Chemical
- Solar
- Nuclear
- Electrical

(ii) Based on propellent used

- liquid propellent,

- Solid Propellent.
- Hybrid Propellent.

(iii) Based on Application

- Space rockets.
- Military rockets
- Weather rockets
- Sustaining rockets

- Based on number of stages.
  - Single stage
  - Multi stage

- Based on size & Range
  - short Range small rockets
  - Long range large rockets.

Propellants:

(i) Mono - Propellants

(ii) Bi-propellants.

Monopropellants:

A liquid propellant which contains both the fuel & oxidizer in a "single chemical".

(Ex): Hydrogen peroxide (H_2O_2)

Hydrazine (N_2H_4)

Bi-propellants:

The fuel and oxidizer are different from each other in its chemical nature.

Ex) Nitrogen Tetroxide (N_2O_4)

Liquid oxygen (Lox)

Feeding systems:

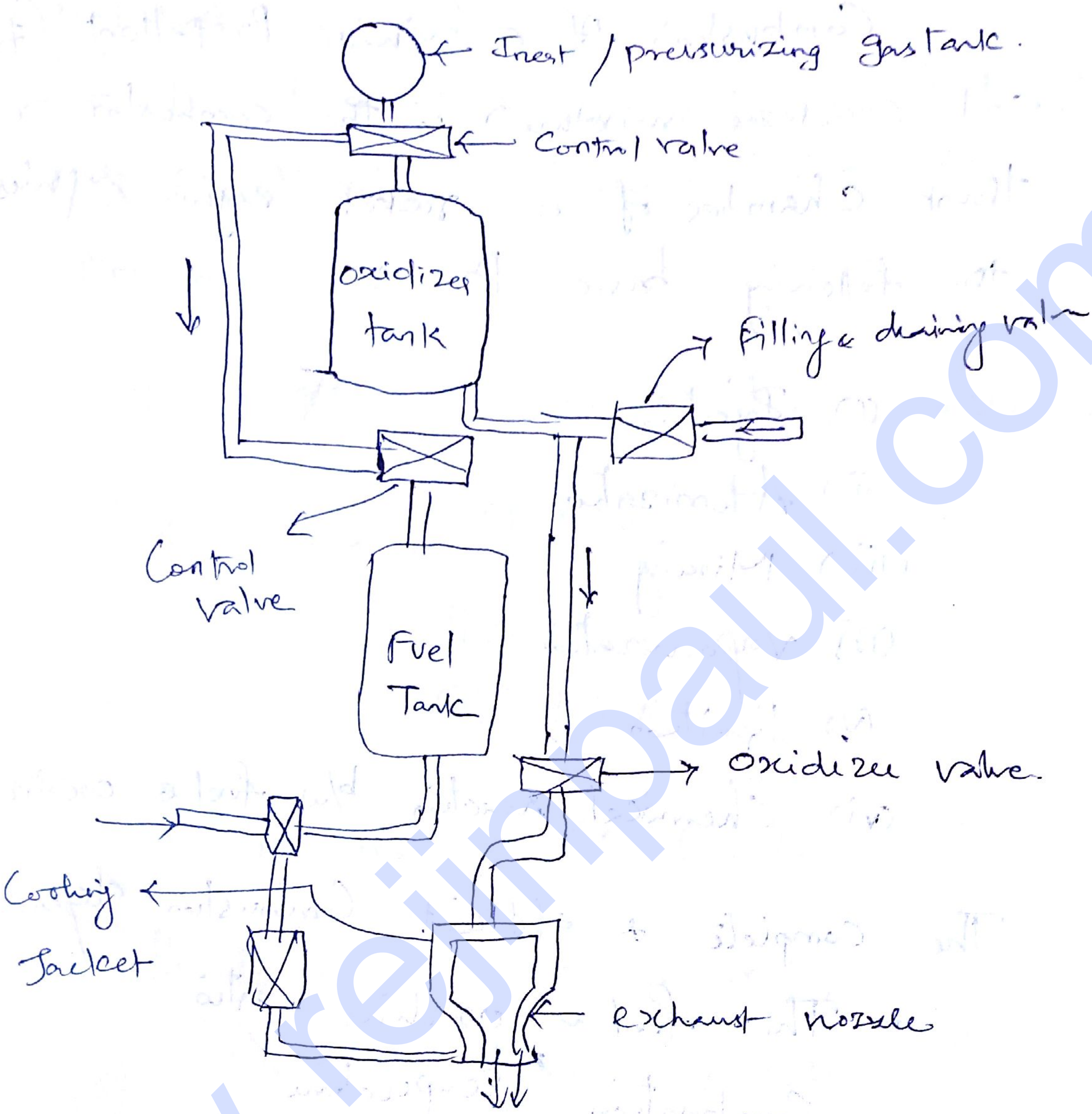
The liquid propellants are fed to combustion chamber at high pressure for better mixing of fuel & oxidizer.

Types:

(i) Gas pressure feed system

(ii) Pump feed system

Gas pressure feed system:



It consists of

(i) Pressurizing (Inert) Gas tank

(ii) Fuel & oxidizer tank

(iii) valves.

(iv) Regulator

(v) Combustion Chamber.

Ignition & Combustion:

Combustion of a liquid Propellant (fuel and oxidizer mixture) in the combustor or thrust chamber of a rocket engine requires the following basic Processes.

(i) Injection
(ii) Atomization
(iii) Mixing
(iv) vapourization
(v) Ignition
(vi) Chemical reaction b/w fuel & oxidizer.

The complete & efficient Combustion depends on

- The fuel & oxidizer ratio
- Combustion temperature
- Time & space available in the C.C
- Degree of atomization & mixing
- fuel injector orifice size
- Ignition at right time.
- The pressure drop b/w Combustion Chamber and fuel line.

Rocket Propulsion theory:

(i) Thrust:

The force that propels the rocket at a given velocity is known as "thrust (or) propulsive force":

$$\text{Thrust } (F) = F_{momentum} + F_{pressure\ difference}$$

WKT,
$$F_{mom} = \dot{m}_p \times C_e$$
$$F_{pr} = P_e A_e - P_a A_e$$
$$= (P_e - P_a) A_e$$

$$\therefore F = \dot{m}_p C_e + (P_e - P_a) A_e$$

When
$$P_e = P_a = F = \dot{m}_p C_e \text{ (or) } \dot{m}_p C_j.$$

$\dot{m}_p$ - mass of the propellant

A_e - nozzle exit area

C_e - nozzle exit velocity

P_e - nozzle exit pressure.

P_a - Ambient pressure.

C_j - jet velocity

(ii) Specific Impulse

Thrust per unit weight flow rate of the propellant.

$$I_{sp} = \frac{F}{W_p} = \frac{\dot{m}_p \times C_j}{\dot{m}_p \times g} = \frac{C_j}{g} \text{ sec.}$$

(iii) Specific Propellant Consumption (SPC)

The weight flow rate of propellant required to produce a thrust of one newton

$$SPC = \frac{W_p}{F} = \frac{\dot{m}_p \times g}{F} = \frac{\dot{m}_p \times g}{\dot{m}_p \times C_j}$$

$$SPC = \frac{g}{C_j} = \frac{1}{I_{sp}}.$$

(iv) Weight flow Coefficient (C_w)

Ratio of the gas/propellant flow rate to the force.

$$C_w = \frac{W_p}{F^*} = \frac{W_p}{P_0 A^*}$$

(v) Thrust Coefficient.

Ratio of Thrust to force.

$$C_F = \frac{F}{F^*} = \frac{F}{P_0 A^*}$$

(vi) Impulse to weight ratio:

It is the ratio of impulse of the rocket to the total weight of the rocket.

$$IWR = \frac{\text{Total Impulse}}{\text{Total weight}} = \frac{I_{total}}{W_{total}}$$

$$I_{Total} = I_{sp} \times W_{total}$$

(vii) Characteristic velocity (C^*):

The ratio b/w effective jet velocity and thrust coefficient.

$$C^* = \frac{C_j}{C_F}$$

Performance study - staging & terminal & Characteristics velocity:

(i) Propulsive efficiency.

It is defined as the ratio of Propulsive Power (or) thrust Power to the engine O/p Power.

$$\eta_P = \frac{\text{Thrust Power}}{\text{engine o/p Power}}$$

Thrust Power $= \dot{m}_p \times C_j \times u$

Engine o/p Power $= \frac{1}{2}\dot{m}_p\,(C_j^2 + u^2)$

$$\therefore \quad \eta_p = \frac{\dot{m}_p\, C_j \times u}{\frac{1}{2}\dot{m}_p\,(C_j^2 + u^2)}$$

$$\boxed{\eta_p = \frac{2\,C_j\,u}{C_j^2 + u^2}}$$

Thermal efficiency:

It is defined as the ratio of engine o/p Power to the Power i/p through fuel.

$$\eta_t = \frac{\frac{1}{2}\dot{m}_p\,(C_j^2 - u^2)}{\dot{m}_p \times C_v}$$

$$\eta_t = \frac{C_j^2 + u^2}{2 \times C_v}$$

Overall efficiency:

$$\eta_o = \frac{\text{Propulsive Power}}{\text{Power i/p through fuel.}}$$

$$\eta_o = \frac{C_j \times u}{CV}$$

$$\boxed{\eta_o = \eta_p \times \eta_t}$$

Applications:

- Military
- Space exploration
- Weather Prediction
- Communication
- Aircraft Propulsion
- Scientific Investigation.

Space flights:

- Man's interest in the heavens bey the skies & the infinite number of stars dates back thousands of year be Chri

- Space fiction describes journeys be the earth on birds and capsules fired by gigantic cannons. Some kind of rockets h also been mentioned.

- On account of very long distances inv in space travel rockets have several stages which are successively dropped after their operations.

- This eliminates the problem of Carry the structures of the "spent up" rockets over long distances.

- Besides this each individual rocke must have the largest proportion of the propellant weight compared to the tota weight.