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Report of ACI-ASCE Committee 326

Shear and Diagonal Tension

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Discussion of this report (three parts) should reach ACI headquarters in triplicate by June 1, 1962, for publication in the September 1962 JOURNAL.

Presents a review of scientific knowledge, engineering practice, and construction experiences regarding shear and diagonal tension in reinforced concrete beams, frames, slabs, and footings. Recommendations for new design procedures are substantiated by extensive test data.

Chapters 1 through 4 deal with background and general principles. Chapters 5 through 7 present the development of new design methods for reinforced concrete members without and with web reinforcement, and for members without and with axial load acting in combination with bending and shear. Chapter 8 deals with slabs and footings including the effect of holes and transfer of moments from columns to slabs.

Esfuerzo Cortante y Tensión Diagonal

Se revisan los conocimientos científicos, ingeniería práctica y experiencias en construcciones relativas al esfuerzo cortante y tensión diagonal en vigas, armaduras (marcos rigidos), losas y cimientos de hormigón armado. Se recomiendan

nuevos procedimientos de diseño comprobados por los datos obtenidos por medio de ensayos extensivos.

Los capítulos 1 al 4 tratan de los antecedentes y principios generales. Los capítulos 5 al 7 presentan el desarrollo de nuevos métodos de diseño para miembros de hormigón armado con y sin refuerzo del alma y para miembros con y sin carga axial combinada con la flexión y el esfuerzo cortante. El capítulo 8 trata de las losas y cimientos incluyendo el efecto causado por agujeros y el traspaso de momentos de las columnas a las losas.

L'Effort Tranchant et la Contrainte Principale

On présente une revue de l'art scientifique, de la pratique du génie et des expériences dans la construction relatives aux efforts tranchants et à la contrainte principale dans les poutres, les portiques, des dalles et les semelles de fondations en béton armé. Les recommendations pour les nouvelles procédés de calcul sont justifées à l'aide de résultats nombreaux d'essais.

Les chapitres 1 à 4 concernent les bases et les principes généraux. Les chapitres 5 à 7 présentent l'évolution de nouvelles methodes de calcul d'éléments en béton armé avec et sans armatures de cisaillement et d'éléments soumis à la flexion simple et composée avec les efforts tranchants. Le chapitre 8 concerne les dalles et les semelles de fondations y compris l'influence des trous et la transmission de moments de flexion des colonnes aux dalles.

Schub- und Hauptzugspannungen

Es wird eine Übersicht gegeben über wissenschaftliche Erkenntnisse, technische Praxis und Bauerfahrungen bezüglich Schubsicherung in Stahlbetonträgern, Rahmen, Platten und Säulenfussplatten. Empfehlunger für neue Berechnungsverfahren werden durch umfassende Versuchsunterlagen erhärtet.

Kapitel 1 bis 4 behandeln die Vorgeschichte und die allgemeinen Grundsätze. Kapitel 5 bis 7 enthalten die Entwicklung von neuen Berechnungsmethoden für Stahlbetonteile ohne und mit Schubbewehrung und für Bauglieder unter Biegung und Schub ohne und mit gleichzeitiger Längskraft. Kapitel 8 behandelt Platten und Säulenfussplatten einschliesslich der Wirkung von Aussparungen und die Übertragung von Momenten von Säulen zu Platten.

ACI-ASCE Committee 326, Shear and Diagonal Tension, was formed in 1950 to develop methods for designing reinforced concrete members to resist shear and diagonal tension consistent with ultimate strength design. Several investigations and test programs were initiated, sponsored and conducted by numerous organizations, including Committee 326, the Reinforced Concrete Research Council, many universities (especially the University of Illinois), the American Iron and Steel Institute, and the Portland Cement Association. Progress reports of Committee work were presented at the ACI 55th annual convention, February 1959, and the 56th convention, March 1960. This three-part report is the culmination of a 10-year study.

CHAPTER 8 — SLABS AND FOOTINGS†

800—Introduction

It is not economically feasible to test numerous slab floor systems to determine the shear strength of the column to slab connection or to determine the effect of a concentrated load. However, the shear strength problem in the vicinity of a concentrated load or reaction may be considered a localized condition involving only a portion of the slab around the loaded area. Consequently, laboratory specimens have generally been square slabs with column stubs at the center of the slab and with supports at the four edges. This specimen approximates the portion of a flat plate floor system which extends from the column out to the region of contraflexure of the slab.

This type of test specimen is similar to a column footing, except for the support conditions. In many respects reinforced concrete column footings are acting as a portion of an inverted flat slab. Therefore, the shear problem of a concentrated load on a slab, of a column to slab connection in a flat slab or flat plate, and of a column connection in a footing, are in reality similar problems. They will be considered simultaneously in this report unless otherwise noted.

801—Review of research

While the first theoretical and experimental investigations of the behavior of reinforced concrete beams was reported during the last years of the nineteenth century,[1] the results of the first extensive study of the shear strength of slabs was published in 1913 when *Talbot*[2] presented his well-known investigation of reinforced concrete footings. Altogether 114 wall footings and 83 column footings were tested to failure. Of the latter, approximately 20 specimens failed in shear. Talbot computed shear stress by the formula

$$v = \frac{V}{4(r + 2d)jd} \qquad \text{(8-1)}$$

in which

v = shear stress
V = shear force
r = side dimension of square column
d = effective depth of slab
jd = internal moment arm of slab

It was found that relatively high values of shear strength were obtained when large percentages of tensile reinforcement were used in the slabs.

† This chapter was developed largely on the basis of the work by Johannes Moe reported in Reference 20 of Section 811.

Talbot's study of reinforced concrete footings was reflected in the design practice of many countries throughout the world.

In 1915, *Graf and Bach*[3] reported a large number of slab tests which were undertaken mainly to study flexural strength. Most of the slabs were loaded simultaneously at eight or more points. A few slabs, loaded at the center only, failed in shear; a portion of the slab having the form of a truncated cone was pushed out underneath the load.

The shear strength of slabs loaded by concentrated loads near supports was studied by *Graf*[4] in 1933. A series of shear tests was carried out on each of three slabs. The results showed clearly that the shear capacity decreased as the load was moved away from the supports. Graf computed shear stress by the formula

$$v = \frac{V}{4\,rt} \qquad (8\text{-}2)$$

where t is the total depth of the slab.

Graf found that the shear strength increased with concrete strength, but at a lower rate than compressive and tensile strength. He suggested that flexural cracking may have some influence on shear strength.

Another series of tests was reported by *Graf*[5] in 1938. Eight very thick slabs, six of which had shear reinforcement, were tested. The slabs, which were approximately 5½-ft square, were supported along all four edges. The thicknesses varied between 12 and 20 in.

Richart and Kluge,[6] in 1939, reported an investigation of reinforced concrete slabs subjected to concentrated loads, which was undertaken to provide information on the design of highway bridge deck slabs. As part of the investigation, seven shear tests were made on two long rectangular slabs with an effective depth of 5.5 in. and a short span of 6 ft 8 in. The loaded areas were circular, 6 and 2 in. in diameter. This report also includes data on shear tests of eighteen 5-ft square slabs simply supported on two edges only. As shown in a recent study by *Elstner and Hognestad*,[7] many of these slabs seem to have failed in flexure, but some shear failures are also recorded. The main purpose of the latter series was to determine the effect of size and shape of the load-bearing area.

While the ultimate shear stresses, as computed by Eq. (8-1), were approximately 0.08 times the cylinder compressive strength f_c', for the long slabs, only about 60 percent of that stress was obtained in the latter series. The authors concluded that the stresses obtained by using Eq. (8-1) are only nominal and arbitrarily chosen, and that an increase in flexural strength of the square slabs would probably have increased shear strength.

Forsell and Holmberg,[8] in 1946, reported on extensive shear tests of slabs carried out from 1926 to 1928. One series of tests was made on circular slabs of sand-cement mortar. The slabs which had diameters of

11.5 to 17 in. were reinforced by spiral reinforcement and the edges were strengthened by heavy steel rings. The thickness of the slabs varied from 1½ to 3¼ in. Shear stresses, assumed to be parabolically distributed across the depth of the slabs, were computed by the formula

$$v = \frac{1.5\,V}{bt} \qquad (8\text{-}3)$$

The critical section determining the perimeter b was taken at a distance of $t/2$ from the edges of the loaded area. The diameter of the supporting ring, which varied between 6 and 12 in., was in many cases so small compared to the thickness of the slab that the base of the frustum of the cone forming at failure extended all the way to the supports. Considerable increase in shear strength due to the influence of the supports is probable under such conditions.

In a second series, twenty-two 4 ft square reinforced concrete slabs were tested. The slabs had a thickness of approximately 4¾ in. All slabs but one were supported along all four edges with the corners tied down. The main variables were concrete strength and manner of loading.

One reinforced concrete slab continuous over three spans was also tested by Forsell and Holmberg. Eleven shear tests were made at different points of this slab. The influence of bending moment on shear strength was clearly demonstrated.

At the University of Illinois, *Newmark, Siess, et al.,*[9,10,11] as part of an extensive investigation of highway bridges, reported a number of shear tests of mortar bridge deck slabs 1¾ in. thick. Their first report includes tests of 15 models of simple span, right angle I-beam bridges. All slabs failed in shear at loads P_{test}, considerably higher than those causing first yielding of the tensile reinforcement P_{yield}. The average value of P_{test}/P_{yield} was as high as 1.8, thus indicating that the ultimate flexural capacities of the slabs probably were almost exhausted at the time of shear failure. The authors stated that the loads at shear failure to a certain degree seemed to be dependent on the same factors as the loads at first yielding.

The second Illinois report describes tests on five skew simple-span I-beam bridges with angles of skew of 30 to 60 deg. Final failure in all cases took place by shear at loads higher than for the right-angle bridges.

In the third Illinois report, tests on three two-span continuous I-beam bridges are reported. Higher shear strengths were found at midspan than over the supports, probably as a result of the direct axial forces in the deck slabs, which acted as flanges in the composite I-beam sections.

Richart,[12] in 1948, reported the results of an extensive investigation on reinforced concrete footings. In all, 24 wall footings and 140 column footings were tested to failure. Of the latter, 128 were 7-ft square, the rest being of rectangular shapes, 6 x 9 ft, and 6 x 10 ft. The major

variables were: amount, strength, bond characteristics, and end anchorage of tensile reinforcement; concrete strength; and effective depth of the footings. Apparently, 106 of the column footings failed in shear.

Richart concluded that shear stress rather than bond stress may frequently be a critical feature in the design of a footing. The shear stress at failure, calculated by Eq. (8-1) at a distance d from the faces of the columns generally varied from less than 0.05 f_c' to 0.09 f_c'. The value of v/f_c' increased consistently as the effective depth of the footing decreased. When high tensile stresses in the flexural reinforcement and extensive cracking of the footings were present, an early failure in shear evidently took place.

Hahn and Chefdeville,[13] in 1951, presented a short description of shear tests on three slabs. The specimens were approximately 7-ft square with thicknesses of 9 to 10 in., and were loaded through a 20 in. square column stub. Two of the slabs were strengthened in shear by heavy structural steel crossheads.

A few shear tests on deck slabs of I-beam bridges were reported by *Thomas and Short*[14] in 1952. One of these deck slabs was prestressed.

Hognestad,[15] in 1953, published the results of a re-evaluation of the shear failures of footings which were reported by Richart. Hognestad recognized the effect of superimposed flexure on ultimate shear strength, and he introduced the ratio $\phi = V_{test}/V_{flex}$ as one of the parameters in his statistical study of the test results. In this ratio, V_{test} is the observed shear force at shear failure, V_{flex} is the shear force at flexural ultimate strength as computed by yield-line theory. He also suggested that shear stress be computed at zero distance around the loaded area because this seemed to give the best measure of the shear strength.

The following ultimate shear strength equation was found to apply within the range of variables covered by Richart's tests

$$v = \frac{V}{\frac{7}{8}\,bd} = \left(0.035 + \frac{0.07}{\phi_o}\right) f_c' + 130 \text{ psi} \qquad (8\text{-}4)$$

where $\phi_o = V/V_{flex}$.

Hognestad indicated that Eq. (8-4) was found to apply for values of r/d, the ratio of column width to effective slab thickness, between 0.88 and 2.63, but is probably unsafe for values of f_c' below 1800 psi. A new design method based on Eq. (8-4) was suggested and compared to the provisions of the 1951 ACI Building Code.

Elstner and Hognestad,[7] also in 1953, reported shear tests of twenty-four 6-ft square and 6 in. thick reinforced concrete slabs. The majority of these slabs were supported along all four edges. The results of these tests, as well as those reported by Forsell and Holmberg,[8] and Richart and Kluge[6] were analyzed and compared to the strengths predicted by Eq. (8-4).

Keefe,[16] in 1954, investigated the effectiveness of a special type of shear reinforcement known as "shearheads." Two pairs of octagonally shaped slabs were tested, one of each pair being furnished with a "shear-head," while the other had no shear reinforcement. The slabs with shearhead had an ultimate shear capacity approximately 40 percent higher than those without.

Elstner and Hognestad,[17] in 1956, reported tests of thirty-eight 6 ft square slabs which were loaded through column stubs in the center, and, with a few exceptions, were supported along all four edges. Twenty-four of these tests were also reported earlier.[7] The major variables were: concrete strength, percentage of flexural tension and compression reinforcement, percentage of shear reinforcement, and size of column. The effect of concentration of the flexural reinforcement was also explored. No effects on the ultimate shear strength were found due to variation in concentration of the tension reinforcement under the column or the amount of compression reinforcement. The new test results indicated that Eq. (8-4) gave unsafe values of the ultimate shear strength for high concrete strengths (4500-7300 psi). By statistical analysis of all of the slabs, except those with shear reinforcement, the following equation was found to be in better agreement with the test results

$$v = \frac{V}{\frac{7}{8}\,bd} = 333 \text{ psi} + 0.046\, f_c'/\phi_o \qquad \text{(8-5)}$$

For the slabs with shear reinforcement, the following equation was suggested on the basis of new tests and the tests reported by Graf[4,5]

$$v = 333 \text{ psi} + 0.046\, f_c'/\phi_o + (q_u - 0.050) f_c' \qquad \text{(8-6)}$$

where

$$q_u = \frac{A_v f_y \sin \alpha}{\frac{7}{8}\, bd\, f_c'} \qquad \text{(8-7)}$$

and

A_v = area of shear reinforcement
f_y = yield point of shear reinforcement
α = inclination of shear reinforcement

Eq. (8-6) indicates that the shear reinforcement is not fully effective.

Whitney,[18] in 1957, presented an ultimate strength theory for shear which is radically different from the earlier approaches to this problem. Whitney based his study on previously reported test results[12,17] of slabs and footings but excluded a number of tests which he believed involved bond failure. The excluded slabs had large amounts of tension reinforcement consisting of closely spaced bars. For the remaining slabs, Whitney assumed that the shear strength is primarily a function of the ultimate resisting moment m of the slab per unit width inside the "pyramid of

rupture," i.e., the frustum of a cone or pyramid with surfaces sloping out in all directions from the column at an angle of 45 deg.

Whitney proposed the following ultimate shear strength equation

$$v = 100 \text{ psi} + 0.75 \frac{m}{d^2} \frac{d}{l_s} \qquad (8\text{-}8)$$

where v is computed at a distance of $d/2$ from the surfaces of the loaded areas, and l_s is the "shear span" which in the case of a slab supported along the edges is taken as the distance between the support and the nearest edge of the loaded area. In the case of a footing with uniform distribution of the reaction, l_s is taken as half of the distance between the edge of the footing and the face of the column.

Since the test results of specimens with relatively high flexural strengths were omitted in the study leading to Eq. (8-8), it can only apply in cases of nearly balanced design, i.e., when ϕ_o is close to unity.

According to Eq. (8-8) the shear strength of a slab can be effectively raised by increasing the amount of flexural reinforcement inside the pyramid of rupture. This also should apply if the increase of reinforcement through the pyramid of rupture is accomplished by shifting tensile reinforcement from outside the "pyramid" to the inside. Hence, Whitney's formula agrees with the 1956 ACI Building Code in that a concentration of the flexural reinforcement in a narrow band across the column is assumed to increase shear strength.

In computing the ultimate shear force V in footings, Whitney subtracted the support reaction inside a distance of $d/2$ from the faces of the column. Most other investigations have subtracted the total support reaction on the base of the "pyramid of rupture."

In a recent study, *Scordelis, Lin, and May*[19] investigated the shear strength of prestressed lift slabs by testing fifteen 6 ft square slab specimens. The slabs were supported along all four edges; twelve of the slabs were prestressed with unbonded cables. Major variables were concrete strength, amount of prestressing, size of steel collars, thickness of slabs, and amount of collar recess.

The ultimate shear strengths were compared to the predictions of Eq. (8-5) and (8-8), and reasonably good agreement was found in both cases. To apply these equations, it was necessary to evaluate the ultimate flexural moment capacities of the slabs. For this purpose it was necessary to make some assumptions regarding the values of the steel stress at ultimate moment and the effect of recesses in the slabs on the ultimate moment capacity. The assumptions made appear reasonable, but they could be questionable.

Moe[20] in 1961, reported tests of forty-three 6 ft square slabs which were similar to the test specimens of Elstner and Hognestad. Moe's principal variables were: effect of openings near the face of the column,

effect of concentration of tensile reinforcement in narrow bands across the column, effect of column size, effect of eccentricity in applied load, and effectiveness of special types of shear reinforcement. He also included a statistical study of 260 slabs and footings tested by earlier investigators.

Moe's work represents a thorough, complete and up-to-date study of the shear strength of slabs based on practically all available data. Major portions of this chapter were taken almost verbatim from Moe's report, a draft copy of which was made available to Committee 326 by the Portland Cement Association.

Some of the more important conclusions arrived at by Moe are:

1. The critical section governing the ultimate shear strength of slabs and footings should be measured along the perimeter of the loaded area.
2. The shear strengths of slabs and footings depend on flexural strength.
3. The triaxial state of stress in the compression zone at the critical section influences the shear strength of that section considerably as described in Section 402 of this report.
4. The shear strength of the concrete is highest when the column size is small compared to the slab thickness.
5. The ultimate shear strength of slabs and footings is predicted with good accuracy by the formula

$$v_u = \frac{V_u}{bd} = \left[15\left(1 - 0.075\,\frac{r}{d}\right) - 5.25\,\phi_o \right] \sqrt{f_c'}$$

where
b = perimeter of the loaded area
d = effective depth of slab
r = side length of square loaded area
V_u = ultimate shear force
V_{flex} = ultimate shear force if flexural failure had occurred
ϕ_o = V_u/V_{flex}

For footings

$$V_u = \left[1 - \left(\frac{r + d}{a}\right)^2 \right] P_u$$

where
P_u = total load on loaded area
a = side length of square footing slab

6. Inclined cracks develop in the slabs at loads as low as 50 percent of the ultimate.
7. Loads 50 percent above the inclined cracking load, sustained for 3 months, did not affect the ultimate shear strength.

8. The effect of openings adjacent to the column may be accounted for by introducing the net value for the perimeter b into the equation in Conclusion 5.

9. Concentration of flexural reinforcement in narrow bands across the column did not increase the shear strength. However, such concentration increased the flexural rigidity of the test slabs, and also increased the load at which yielding began in the tension reinforcement.

10. Some increase in shear strength can be obtained by shear reinforcement. However, the anchorage of such reinforcement in the compression zone seems to be problematical, therefore the use of shear reinforcement in thin slabs was not recommended.

11. In cases of moment transfer between square columns and slabs, test results indicate it is safe to assume that one-third of the moment is transferred through vertical shear stresses at the perimeter of the loaded area distributed in proportion to the distance from the centroidal axis of the loaded area. Maximum shear stress due to the combined action of vertical load and moment should not exceed the value expressed by the equation in Conclusion 5.

802—Review of design specifications

The limited nature of knowledge previously available regarding the mechanism of failure in shear of slabs under concentrated loads is clearly reflected in the standard specifications of various countries. Quite different rules are applied to determine the critical shear or inclined tensile stresses, and the allowable stresses vary considerably.

In the *United States* the first standard specifications,[21] which were prepared by a Joint Committee appointed by a number of professional societies, stipulated an allowable shear stress in pure shear equal to 0.06 f_c'. The shear stress should be computed by $v = V/bt$, where the critical section was taken along the perimeter b of the loaded area, and t is the total slab thickness. In the revised version of 1917[22] it is also required that diagonal tension requirements be met, but no rules were given for determining diagonal tension stress.

The *ACI Standard of 1916*[23] allowed a stress in pure shear equal to 0.075 f_c' computed along the periphery of the loaded area.

In the *ACI Standard of 1920*[24] a clear distinction was made between the following two possible types of shear failure:

(a) A pure shear failure controlled by the allowable shear stress computed at zero distance from the periphery, and stipulated at 0.10 f_c'.

(b) A so-called "diagonal tension failure" controlled by shear stress computed by the formula $v = V/bjd$ at a distance of $d/2$ from the periphery, and limited to 0.035 f_c'.

In the report of the *Joint Committee of 1924*[25] it was specified that shear stress should be computed at a distance of (t — 1½ in.) from the periphery of the loaded area, and the allowable shear stress was given by

$$v = 0.02 f_c' (1 + n) \leq 0.03 f_c' \qquad (8\text{-}9)$$

where n is the ratio of the area of the reinforcing steel crossing directly through the loaded area (column or column capital) to the total area of tensile reinforcement.

The Report of 1924 was also adopted by the American Concrete Institute as a standard, and only minor changes have been made later with respect to shear and diagonal tension in slabs and footings.

The *1956 ACI Building Code (318-56)*[26] allowed the following shear stresses, computed on a critical section at a distance d beyond the periphery of the loaded area:

$0.03f_c' \leq$ 100 psi if more than 50 percent of the tensile reinforcement required for bending passes through the periphery
$0.025f_c' \leq$ 85 psi when only 25 percent of the tensile reinforcement passes through the periphery
$0.03f_c' \leq$ 75 psi for footings

In *Germany* a completely different approach to the design problem of shear in slabs has been practiced. In determining shear as well as flexural stresses, slab strips of certain widths are assumed. The widths given for shear computations are different from those in moment, and the widths also vary with the position of load on the slab. The *German Specification DIN 1045 of 1943*[27] gives the following formulas for the effective slab strip width in shear

$$b_1 = r + 2s \quad \text{and} \quad b_2 = \tfrac{1}{3}\left(l + \frac{r + 2s}{2}\right) \qquad (8\text{-}10)$$

where s is the thickness of a load-distributing layer on the top of the slab and l is the span of the slab.

The larger of the values b_1 and b_2 can be used. In the case of a load close to one of the supported edges, b shall be taken as $r + 5t$.

In some countries, such as Norway, a combination of the American and the German practice has been used. The *Norwegian Standard Specifications of 1939*[28] assumed the shearing stresses to be evenly distributed around the loaded area at a distance of $2d/3$ from the periphery. It is however, also necessary to consider a strip of the slab of a certain specified width as a beam and check the shearing stresses in this beam strip. If a load is placed close to one of the supported edges of a slab, this last check frequently gives the highest shearing stresses.

In the *British Code of Practice (CP114)*[29] the shear stresses in flat slabs are computed at a distance of $d/2$ from the periphery of the loaded area, while in footings the distance is taken equal to d.

803—Development of design recommendations

The evolution of the current concepts of shear action were pointed out in the review of research in Section 801. Three principal variables affect shear strength. They are: concrete strength, the relationship between size of loaded area and slab thickness, and the relationship between shear and moment in the vicinity of the loaded area. The most recent study by Moe led to the empirical ultimate strength design equation

$$v_u = \frac{V_u}{bd} = \left[15\left(1 - 0.075 \frac{r}{d} \right) - 5.25\, \phi_o \right] \sqrt{f_c'} \quad \text{..........(8-11)}$$

where

v_u = permissible ultimate shear stress
V_u = ultimate shear capacity
b = periphery around the loaded area
d = effective depth of the slab
r = side dimension of the loaded area
ϕ_o = V_u/V_{flex}
V_{flex} = ultimate shear force for flexural failure

Comparison of the equation with available data for 198 tests is presented in Table 8-1 of Section 810. The comparison is good as shown by the average ratio of V_{test} to V_{calc} and the standard deviation for each test series. The equation seems to be the best that has been developed to correlate laboratory shear tests of slabs and footings. However, the step from a research equation with limited ranges of applicability to a generally applicable practical design procedure is not an easy one in this particular case.

It was stated previously that shear failure is a local one, involving a portion of the slab structure around the loaded area. Therefore the simple slab specimens used in laboratory tests represent a portion of the slab from the loaded area out to the region of contraflexure. However, because of our limited knowledge concerning the distribution of moments in slab structures at loads near ultimate strength, it is difficult to determine which portion of the slab structure is involved in the shear failure.

Of course, this portion of slab area must be established only to determine the value of ϕ_o in Moe's equation. The variable ϕ_o is dependent on the ultimate flexural capacity of this portion. For simple laboratory specimens computation of flexural capacity is an easy task with the aid of the yield-line theory. For practical design cases, this is more difficult, and the yield-line theory has not yet become generally used in American practical design for flexure.

The variable ϕ_o is the ratio of shear capacity to flexural capacity. For balanced failure, shear capacity and flexural capacity are reached simultaneously, so that ϕ_o equals unity. If the slab fails in shear, ϕ_o is less

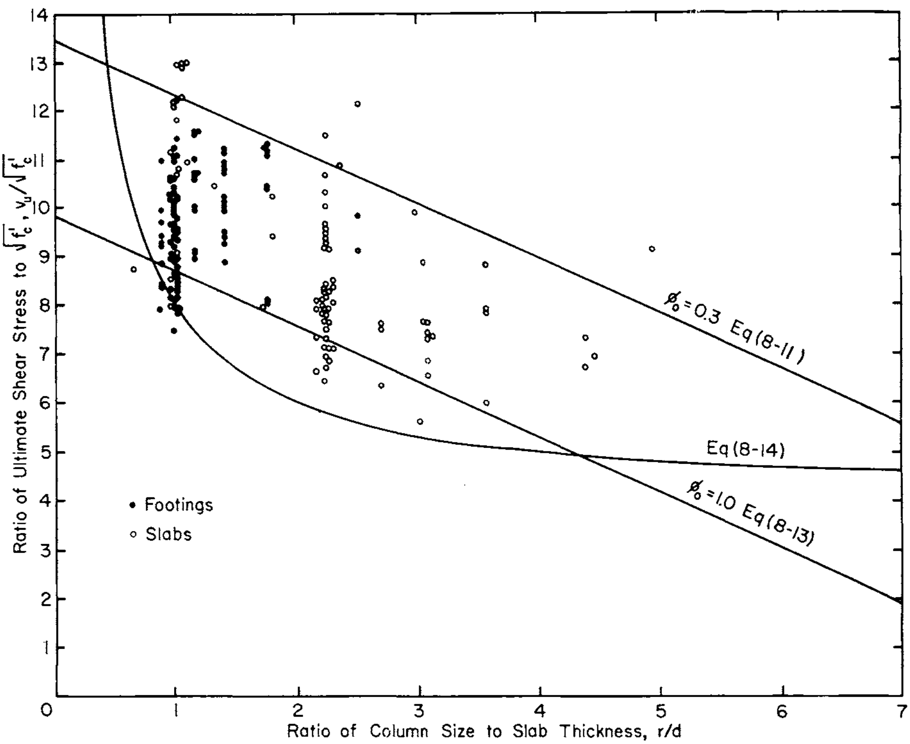


Fig. 8-1—Comparisons of design equations and test data

than unity. Since Moe's equation is only applicable to shear failures, ϕ_o cannot exceed unity.

As ϕ_o decreases, the shear strength of a slab increases. At first glance it would appear that increasing flexural capacity, which in turn decreases ϕ_o would be an efficient way of increasing shear capacity. However, substituting $\phi_o = V_u/V_{flex}$ into Moe's equation leads to

$$v_u = \frac{V_u}{bd} = \frac{15(1 - 0.075\, r/d)\sqrt{f_c'}}{1 + (5.25\, bd\sqrt{f_c'}/V_{flex})} \qquad (8\text{-}12)$$

In Eq. (8-12) it is seen that V_{flex} must be increased substantially to affect a small increase in shear capacity. Although ϕ_o is an important variable in the shear stress equation, the interrelationship of ϕ_o, V_u, and V_{flex} is such that it is uneconomical to control shear capacity by the flexural capacity.

Generally speaking, the laboratory specimens were designed to fail in shear. The term ϕ_o was an important variable in the laboratory tests and it must be considered when laboratory test results are analyzed.

TABLE 8-1—COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Richart's footings — Series 1							
105a	14.00	14.00	58.6	0.851	432	466	1.078
105b	14.00	14.00	48.9	0.760	379	359	0.948
106a	14.00	14.00	61.2	0.917	435	467	1.072
106b	14.00	14.00	60.2	0.906	430	421	0.979
107a	14.00	14.00	60.2	0.924	425	421	0.989
107b	14.00	14.00	58.4	0.908	417	378	0.906
108a	14.00	14.00	64.5	1.074	416	422	1.014
108b	14.00	14.00	57.3	1.003	386	400	1.036
109a	14.00	14.00	54.5	0.576	463	478	1.031
109b	14.00	14.00	55.5	0.583	470	409	0.868
110a	14.00	14.00	56.7	0.626	470	444	0.945
110b	14.00	14.00	51.9	0.592	438	462	1.053
111a	14.00	14.00	54.9	0.623	456	427	0.935
111b	14.00	14.00	58.3	0.648	479	509	1.063
112a	14.00	14.00	58.9	0.505	518	427	0.823
112b	14.00	14.00	53.1	0.471	474	462	0.974
109Ra	14.00	14.00	63.8	0.711	507	511	1.007
109Rb	14.00	14.00	64.6	0.718	512	489	0.953
110Ra	14.00	14.00	56.2	0.600	472	452	0.955
110Rb	14.00	14.00	59.2	0.621	492	527	1.070
						Average =	0.985
						Coefficient of variation =	0.068
Richart's footings — Series II							
201a	14.00	10.00	51.6	0.527	308	274	0.888
201b	14.00	10.00	51.7	0.527	308	312	1.013
202a	14.00	12.00	48.9	0.587	348	379	1.088
202b	14.00	12.00	46.7	0.570	335	362	1.078
Richart's footings — Series II (cont.)							
203a	14.00	14.00	51.3	0.687	413	373	0.904
203b	14.00	14.00	44.9	0.631	372	338	0.907
204a	14.00	12.00	50.8	0.566	385	362	0.937
204b	14.00	12.00	50.4	0.453	383	362	0.944
205a	14.00	14.00	47.5	0.498	420	409	0.974
205b	14.00	14.00	49.1	0.508	431	409	0.949
206a	14.00	16.00	54.0	0.600	526	454	0.862
206b	14.00	16.00	54.0	0.583	508	532	1.047
207a	14.00	8.00	65.2	0.595	289	335	1.158
207b	14.00	8.00	63.5	0.584	284	319	1.123
208a	14.00	10.00	63.3	0.706	344	328	0.950
208b	14.00	10.00	62.0	0.697	339	349	1.029
209a	14.00	12.00	53.7	0.756	353	398	1.126
209b	14.00	12.00	48.6	0.700	326	380	1.163
210a	14.00	10.00	65.7	0.611	376	413	1.099
210b	14.00	10.00	65.7	0.608	373	367	0.985
211a	14.00	12.00	60.5	0.674	412	434	1.051
211b	14.00	12.00	65.3	0.708	437	470	1.073
212a	14.00	14.00	62.4	0.779	478	462	0.966
212b	14.00	14.00	64.0	0.792	487	444	0.910
213a	14.00	8.00	67.1	0.605	296	317	1.069
213b	14.00	8.00	67.5	0.606	297	317	1.064
214a	14.00	10.00	69.1	0.745	368	422	1.148
214b	14.00	10.00	70.3	0.766	372	440	1.180
215a	14.00	12.00	71.1	0.887	431	434	1.007
215b	14.00	12.00	65.0	0.840	405	434	1.070
216a	14.00	8.00	67.5	0.606	297	335	1.126
216b	14.00	8.00	66.8	0.603	295	335	1.134

TABLE 8-1 (cont.)—COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Richart's footings — Series II (cont.)							
217a	14.00	10.00	61.0	0.689	335	349	1.042
217b	14.00	10.00	68.8	0.744	367	422	1.151
218a	14.00	12.00	64.3	0.835	402	434	1.079
218b	14.00	12.00	66.5	0.852	412	398	0.965
						Average =	1.034
						Coefficient of variation =	0.088
Richart's footings — Series III-IV							
304a	14.00	14.00	58.6	0.854	431	466	1.080
304b	14.00	14.00	48.9	0.763	378	359	0.949
305a	14.00	14.00	59.3	0.864	434	498	1.145
305b	14.00	14.00	60.6	0.875	441	496	1.125
307a	14.00	14.00	57.6	0.785	441	462	1.048
307b	14.00	14.00	61.7	0.820	463	492	1.063
308a	14.00	14.00	61.8	0.936	434	476	1.098
308b	14.00	14.00	61.1	0.929	431	400	0.928
309a	14.00	14.00	62.2	0.925	440	421	0.958
309b	14.00	14.00	59.7	0.902	428	400	0.934
310a	14.00	14.00	64.1	0.840	475	533	1.121
310b	14.00	14.00	62.6	0.827	468	467	0.997
312a	14.00	14.00	60.2	0.888	535	455	1.045
312b	14.00	14.00	57.1	0.859	419	333	0.795
314a	14.00	14.00	56.6	0.809	429	489	1.139
314b	14.00	14.00	62.3	0.849	460	413	0.898
315a	14.00	14.00	53.6	0.928	378	333	0.880
315a	14.00	14.00	57.1	0.963	394	410	1.038
316a	14.00	14.00	62.4	0.946	436	467	1.070
316b	14.00	14.00	65.7	0.976	451	444	0.985
Richart's footings — Series III-IV (cont.)							
317a	14.00	14.00	60.1	0.887	434	432	0.994
317b	14.00	14.00	61.0	0.896	438	462	1.053
319a	14.00	14.00	61.6	0.844	456	422	0.924
319b	14.00	14.00	59.2	0.823	443	480	1.083
321a	14.00	14.00	60.4	0.876	439	431	0.980
321b	14.00	14.00	61.9	0.888	447	459	1.026
323a	14.00	14.00	58.0	0.876	421	434	1.028
323b	14.00	14.00	61.3	0.907	438	454	1.036
324a	14.00	14.00	62.4	0.863	457	439	0.961
324b	14.00	14.00	63.3	0.870	460	448	0.969
326a	14.00	14.00	55.4	0.828	414	455	1.099
326b	14.00	14.00	61.8	0.887	446	461	1.032
403a	14.00	14.00	58.6	0.989	418	391	0.935
403b	14.00	14.00	44.3	0.765	342	320	0.935
						Average =	1.010
						Coefficient of variation =	0.081
Richart's footings — Series V-VII							
501a	14.00	10.00	60.7	0.682	335	365	1.089
501b	14.00	10.00	61.1	0.683	336	352	1.046
502a	14.00	16.00	59.4	0.959	478	490	1.025
502b	14.00	16.00	57.3	0.939	466	511	1.096
503a	14.00	16.00	59.6	0.959	479	518	1.082
503b	14.00	16.00	59.0	0.955	476	486	1.022
504a	14.00	10.00	60.1	0.644	334	299	0.894
504b	14.00	10.00	61.1	0.668	339	322	0.948
505a	14.00	16.00	60.6	1.031	467	479	1.025
505b	14.00	16.00	61.1	1.036	469	459	0.978

TABLE 8-1 (cont.) — COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Richart's footings — Series II (cont.)							
506a	14.00	16.00	57.9	1.005	453	437	0.965
506b	14.00	16.00	61.8	1.042	473	437	0.924
701a	21.00	8.00	62.1	0.759	336	382	1.137
701b	21.00	8.00	59.6	0.741	326	395	1.208
702a	21.00	12.00	46.0	0.740	424	376	0.885
702b	21.00	12.00	53.7	0.976	428	440	1.026
						Average =	1.022
						Coefficient of variation =	0.087
Elstner-Hognestad slabs							
A-1a	10	4.63	45.2	0.840	68	68	0.996
A-1b	10	4.63	60.5	0.967	84	82	0.977
A-1c	10	4.63	64.9	1.000	88	80	0.908
A-1d	10	4.63	73.1	1.061	95	79	0.833
A-1e	10	4.63	54.2	0.913	78	80	1.025
A-2a	10	4.50	44.5	0.578	76	75	0.989
A-2b	10	4.50	53.2	0.605	89	90	1.007
A-2c	10	4.50	73.7	0.705	117	105	0.900
A-7b	10	4.50	63.6	0.655	104	115	1.107
A-3a	10	4.50	43.0	0.514	76	80	1.053
A-3b	10	4.50	57.3	0.499	102	100	0.981
A-3c	10	4.50	62.0	0.510	110	120	1.093
A-3d	10	4.50	70.8	0.538	123	123	0.998
A-4	14	4.63	61.6	1.041	98	90	0.920
A-5	14	4.50	63.5	0.720	124	120	0.971
A-6	14	4.50	60.2	0.567	129	112	0.865
A-7	10	4.50	64.3	0.939	87	90	1.028
A-8	14	4.50	56.4	0.964	94	98	1.071

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Elstner-Hognestad slabs (cont.)							
A-11	14	4.50	61.3	0.749	117	119	1.018
A-12	14	4.50	64.2	0.764	121	119	0.982
B-9	10	4.50	79.8	0.809	119	113	0.958
B-11	10	4.50	44.3	0.502	78	74	0.941
B-14	10	4.50	85.6	0.672	138	130	0.940
						Average =	0.980
						Coefficient of variation =	0.067
Graf slabs							
1362	10.8	10.66	47.6	0.682	242	262	1.082
1375	10.8	18.62	47.5	0.676	449	373	0.832
Forsell-Holmberg slabs							
1	4.34	3.98	43.6	0.342	36	41	1.147
2	4.34	4.37	43.6	0.442	38	40	1.042
3	4.34	4.17	43.6	0.455	36	39	1.071
4	4.34	4.33	43.6	0.477	37	40	1.069
5	4.34	4.37	43.6	0.467	38	45	1.182
6	4.34	4.21	43.6	0.395	37	41	1.103
7	4.34	4.17	43.6	0.294	39	42	1.089
8	0.00	4.37	52.1	—	—	26	—
9	0.00	4.21	52.1	—	—	26	—
10	10.90†	4.09	52.1	0.579	49	42	0.855
11	4.34‡	4.42	52.1	0.783	78	63	0.804
12	4.34‡	4.26	52.1	0.840	72	60	0.822
14	10.00§	4.26	49.6	0.501	41	46	1.128
15	4.34	4.33	49.6	0.529	41	40	0.962

TABLE 8-1 (cont.)—COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Forsell-Holmberg slabs (cont.)							
16	4.34	4.26	49.6	0.521	41	43	1.066
17	4.34‡	4.26	49.6	0.672	75	66	0.879
18	4.34	4.42	49.6	0.539	42	42	1.010
19	4.34	4.37	57.1	0.596	46	36	0.784
20	10.00§	4.33	57.1	0.568	47	57	1.230
21	10.00§	3.94	57.1	0.517	42	55	1.288
22	4.34	4.26	57.1	0.583	45	45	0.998
						Average =	1.028
						Coefficient of variation =	0.145
Scordelis-Lin-May slabs							
S-1	13.00	4.25	53.0	0.684	93	105	1.124
S-2	13.00	4.25	63.7	0.728	109	109	1.001
S-4	13.00	2.63	62.9	0.630	53	80	1.517
S-5	13.00	3.00	54.2	0.797	50	60	1.193
S-6	13.00	3.63	68.5	0.928	79	79	0.994
S-7	13.00	4.38	53.7	0.690	98	122	1.234
S-8	13.00	3.63	66.0	0.831	82	100	1.210
S-9	16.00	3.63	66.2	0.901	82	105	1.286
S-10	16.00	3.63	68.3	0.840	89	118	1.321
S-11	13.00	7.63	71.6	0.985	225	225	1.001
S-12	13.00	5.63	70.1	0.820	166	172	1.030
S-13	13.00	3.63	72.3	0.595	107	109	1.017
S-14	13.00	5.63	69.3	0.928	153	168	1.100
S-15	13.00	3.63	71.5	0.701	98	120	1.219
						Average =	1.161
						Coefficient of variation =	0.152

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Moe's slabs							
R-1	††	4.50	63.2	0.882	87.0	88.5	0.973
R-2	6.00	4.50	62.0	0.601	64.5	70.0	1.085
S5-60	8.00	4.50	56.7	0.836	70.3	77.0	1.093
S6-60	10.00‡‡	4.50	55.8	0.897	78.2	72.0	0.921
S7-60	12.00‡‡	4.50	57.8	0.952	87.3	94.8	1.084
S5-70	8.00	4.50	57.8	0.751	75.3	85.0	1.128
S6-70	10.00‡‡	4.50	59.3	0.832	86.7	85.0	0.980
Elstner-Hognestad slabs (concentrated tensile reinforcement)							
A-9	10.00	4.50	65.8	0.704	104.2	100.0	0.959
A-10	14.00	4.50	65.6	0.779	122.5	110.0	0.897
Moe's slabs (concentrated tensile reinforcement)							
S1-60	10.00	4.50	58.1	0.911	80.7	87.5	1.084
S2-60	10.00	4.50	56.6	0.881	80.1	80.0	0.998
S3-60	10.00	4.50	57.3	0.902	80.0	81.8	1.021
S4-60	10.00	4.50	58.8	0.976	78.0	75.0	0.961
S1-70	10.00	4.50	59.6	0.832	87.1	88.2	1.011
S3-70	10.00	4.50	60.7	0.840	88.3	85.0	0.962
S4-70	10.00	4.50	71.4	0.943	97.0	84.0	0.865
S4A-70	10.00	4.50	54.5	0.930	74.7	70.0	0.936

† Rectangular load; area $b = 25.60$ in.

‡ Two load points $b = 8r$.

§ Line load $b = 2r$.

†† Column 6 x 18 in.

‡‡ Square steel plate.

TABLE 8-1 (cont.)—
COMPARISON OF TEST DATA WITH MOE'S EQ. (8-11)

Slab No.	r, in.	d, in.	b_o, in.	$\sqrt{f_c'}$	ϕ_o	V_{calc}, kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
Moe's slabs (slabs with openings)								
H1	10.00	4.50	40.00	61.5	1.000	80.2	83.5	1.041
H2	10.00	4.50	34.75	60.2	0.909	72.6	74.0	1.018
H3	10.00	4.50	29.75	58.6	0.813	64.5	73.0	1.130
H4	10.00	4.50	29.75	61.1	0.830	66.5	65.1	0.978
H5	10.00	4.50	24.60	60.2	0 724	57.9	56.1	0.968
H6	10.00	4.50	20.00	64.2	0.549	52.5	55.2	1.050
H7	10.00	4.50	36.14	60.4	0.931	74.4	70.1	0.941
H8	10.00	4.50	32.28	63.8	0.890	71.8	70.1	0.976
H9	10.00	4.50	36.40	59.1	0.924	73.6	70.3	0.955
H10	10.00	4.50	39.44	60.1	0.989	79.0	75.1	0.949
H11	10.00	4.50	40.00	61.5	1.000	80.2	76.1	0.948
H12	10.00	4.50	40.00	66.1	No slab action		60.4	—
H13	10.00	4.50	40.00	59.7	No slab action		45.1	—
H14	10.00	4.50	35.00	61.6	Flexure failure		56.8	—
H15	10.00	4.50	35.00	58.2	0.902	71.6	74.6	1.042
							Average =	0.992
							Coefficient of variation =	0.057

However, in practical design, ϕ_o is not an important variable because the shear capacity of the slab should exceed its flexural capacity, that is, ϕ_o should be at least equal to unity. Therefore ϕ_o may be eliminated from Moe's Eq. (8-11) by substituting $\phi_o = 1.0$

$$v_u = \frac{V_u}{bd} = \left(9.75 - 1.125 \frac{r}{d} \right) \sqrt{f_c'} \qquad \text{(8-13)}$$

Fig. 8-1 shows test data in terms of the parameters $v_u/\sqrt{f_c'}$ versus r/d. The data are compared to Moe's general equation which produces a family of straight lines having ϕ_o as a parameter. The values of $\phi_o =$ 0.30 and $\phi_o = 1.00$ cover the limits of the test data. It is seen that Eq. (8-13) is conservative even when ϕ_o is less than 1.00. Unfortunately, however, Eq. (8-13) is not satisfactory over the full range of variables encountered in practical design.

If load is applied to a slab over a very small area, the perimeter b and the ratio r/d will be very small. According to Eq. (8-13) the ultimate shear stress v_u will approach $9.75\sqrt{f_c'}$; but the ultimate load capacity V_u will approach zero. This cannot be supported by logical reasoning. A thick slab can carry substantial load without failing in shear even if the load is applied over a very small area.

Likewise the equation obviously does not apply for very large values of r/d since it predicts $v_u = 0$ when $r/d = 8.67$. Drop panels or wall loads can give values of r/d exceeding 8.67. It is not reasonable to expect

that slabs with drop panels or wall loads should have no shear resistance at all.

In the case of a continuous slab supported on a wall, the ratio r/d can, for the purpose of this discussion, be assumed to be infinite. The slab will have comparatively little slab action in the vicinity of the wall and will tend to behave like a wide, shallow beam. Therefore, as r/d approaches infinity, the value of v_u would approach the corresponding shear strength of a beam, which would be $1.9\sqrt{f_c'}$ plus a small term depending on the M/Vd ratio. In view of recent tests by Diaz de Cossio[30] showing the effect of ratio of width to depth on the shear strength of beams, as well as available results of tests on slabs, it appears that the shear strength v_u for slabs with large r/d ratios approaches a value substantially in excess of $1.9\sqrt{f_c'}$. It is consistent with available test results to take the shearing resistance of slabs as approaching the limiting value of $4.0\sqrt{f_c'}$ for large ratios of r/d when two-way slab action is present.

Therefore, the design expression for two-way slab action must satisfy the following conditions:

1. The ultimate shear stress v_u shall be a function of $\sqrt{f_c'}$ and r/d.
2. As r/d approaches zero, the ultimate shear load capacity V_u approaches a finite value.
3. Therefore, when r/d approaches zero, v_u approaches infinity.
4. When r/d approaches infinity, v_u approaches $4.0\sqrt{f_c'}$.
5. The shear stress v_u must decrease continuously to $4.0\sqrt{f_c'}$ as r/d increases.

The above conditions can be satisfied by a hyperbolic equation of the form $v_u = (Ad/r + B)\sqrt{f_c'}$, which for a conservative fit of the test data, gives

$$v_u = 4\left(\frac{d}{r} + 1\right)\sqrt{f_c'} \qquad \text{(8-14)}$$

Eq. (8-14) is plotted on Fig. 8-1 where it can be compared with Moe's Eq. (8-13) and with the test results given in Table 8-2.

The shear load capacity V_u can be evaluated from

$$V_u = v_u\, bd \qquad \text{(8-15)}$$

where b is the periphery at the edge of the loaded area.

Thus, for a square column, $b = 4r$, so that V_u can be expressed as

$$V_u = 16\, d^2\left(\frac{r}{d} + 1\right)\sqrt{f_c'} \qquad \text{(8-15a)}$$

TABLE 8-2 — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Richart's footings — Series I						
105a	14.00	14.00	1.00	594	58.6	10.14
105b	14.00	14.00	1.00	458	48.9	9.37
106a	14.00	14.00	1.00	595	61.2	9.72
106b	14.00	14.00	1.00	537	60.2	8.92
107a	14.00	14.00	1.00	537	60.2	8.92
107b	14.00	14.00	1.00	480	58.4	8.22
108a	14.00	14.00	1.00	538	64.5	8.31
108b	14.00	14.00	1.00	510	57.3	8.90
109a	14.00	14.00	1.00	609	54.5	11.17
109b	14.00	14.00	1.00	520	55.5	9.37
110a	14.00	14.00	1.00	566	56.7	9.98
110b	14.00	14.00	1.00	588	51.9	11.33
111a	14.00	14.00	1.00	543	54.9	9.89
111b	14.00	14.00	1.00	649	58.3	11.13
112a	14.00	14.00	1.00	543	58.9	9.22
112b	14.00	14.00	1.00	588	53.1	11.07
109Ra	14.00	14.00	1.00	650	63.8	10.19
109Rb	14.00	14.00	1.00	623	64.6	9.64
110Ra	14.00	14.00	1.00	574	56.2	10.21
110Rb	14.00	14.00	1.00	672	59.2	11.35
Richart's footings — Series II						
201a	14.00	10.00	1.40	488	51.6	9.46
201b	14.00	10.00	1.40	556	51.7	10.75
202a	14.00	12.00	1.17	563	48.9	11.51
202b	14.00	12.00	1.17	563	46.7	11.52
203a	14.00	14.00	1.00	475	51.3	9.26
203b	14.00	14.00	1.00	430	44.9	9.58
Richart's footings — Series II (cont.)						
204a	14.00	12.00	1.17	538	50.8	10.59
204b	14.00	12.00	1.17	538	50.4	10.67
205a	14.00	14.00	1.00	520	47.5	10.95
205b	14.00	14.00	1.00	520	49.1	10.59
206a	14.00	16.00	0.87	505	54.0	9.35
206b	14.00	16.00	0.87	593	54.0	10.98
207a	14.00	8.00	1.75	747	65.2	11.46
207b	14.00	8.00	1.75	710	63.5	11.18
208a	14.00	10.00	1.40	584	63.3	9.23
208b	14.00	10.00	1.40	623	62.0	10.05
209a	14.00	12.00	1.17	592	53.7	11.02
209b	14.00	12.00	1.17	565	48.6	11.63
210a	14.00	10.00	1.40	736	65.7	11.20
210b	14.00	10.00	1.40	654	65.7	9.95
211a	14.00	12.00	1.17	645	60.5	10.66
211b	14.00	12.00	1.17	699	65.3	10.70
212a	14.00	14.00	1.00	588	62.4	9.42
212b	14.00	14.00	1.00	566	64.0	8.84
213a	14.00	8.00	1.75	705	67.1	10.51
213b	14.00	8.00	1.75	705	67.5	10.44
214a	14.00	10.00	1.40	754	69.1	10.91
214b	14.00	10.00	1.40	785	70.3	11.17
215a	14.00	12.00	1.17	645	71.1	9.07
215b	14.00	12.00	1.17	645	65.0	9.92
216a	14.00	8.00	1.75	747	67.5	11.07
216b	14.00	8.00	1.75	747	66.8	11.18
217a	14.00	10.00	1.40	623	61.0	10.21
217b	14.00	10.00	1.40	754	68.8	10.96

TABLE 8-2 (cont.) — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Richart's footings — Series V-VII (cont.)						
218a	14.00	12.00	1.17	645	64.3	10.03
218b	14.00	12.00	1.17	592	66.5	8.90
Richart's footings — Series III-IV						
304a	14.00	14.00	1.00	594	58.6	10.14
304b	14.00	14.00	1.00	458	48.9	9.37
305a	14.00	14.00	1.00	634	59.3	10.69
305b	14.00	14.00	1.00	632	60.6	10.43
307a	14.00	14.00	1.00	588	57.6	10.21
307b	14.00	14.00	1.00	627	61.7	10.16
308a	14.00	14.00	1.00	507	61.8	8.20
308b	14.00	14.00	1.00	510	61.1	8.35
309a	14.00	14.00	1.00	537	62.2	8.63
309b	14.00	14.00	1.00	510	59.7	8.54
310a	14.00	14.00	1.00	679	64.1	10.59
310b	14.00	14.00	1.00	595	62.6	9.50
312a	14.00	14.00	1.00	580	60.2	9.63
312b	14.00	14.00	1.00	424	57.1	7.43
314a	14.00	14.00	1.00	623	56.6	11.01
314b	14.00	14.00	1.00	527	62.3	8.46
315a	14.00	14.00	1.00	424	53.6	7.91
315b	14.00	14.00	1.00	521	57.1	9.12
316a	14.00	14.00	1.00	595	62.4	9.54
316b	14.00	14.00	1.00	566	65.7	8.61
317a	14.00	14.00	1.00	551	60.1	9.17
317b	14.00	14.00	1.00	588	61.0	9.64
319a	14.00	14.00	1.00	538	61.6	8.73
319b	14.00	14.00	1.00	611	59.2	10.32

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Richart's footings — Series III-IV (cont.)						
321a	14.00	14.00	1.00	501	60.4	8.29
321b	14.00	14.00	1.00	502	61.9	8.11
323a	14.00	14.00	1.00	481	58.0	8.29
323b	14.00	14.00	1.00	483	61.3	7.88
324a	14.00	14.00	1.00	529	62.4	8.48
324b	14.00	14.00	1.00	530	63.3	8.37
326a	14.00	14.00	1.00	499	55.4	9.01
326b	14.00	14.00	1.00	502	61.8	8.12
403a	14.00	14.00	1.00	459	58.6	7.83
403b	14.00	14.00	1.00	446	44.3	10.07
Richart's footings — Series V-VII						
501a	14.00	10.00	1.40	650	60.7	10.71
501b	14.00	10.00	1.40	627	61.1	10.26
502a	14.00	16.00	0.87	545	59.4	9.18
502b	14.00	16.00	0.87	569	57.3	9.93
503a	14.00	16.00	0.87	578	59.6	9.70
503b	14.00	16.00	0.87	542	59.0	9.19
504a	14.00	10.00	1.40	532	60.1	8.85
504b	14.00	10.00	1.40	573	61.1	9.38
505a	14.00	16.00	0.87	534	60.6	8.81
505b	14.00	16.00	0.87	512	61.1	8.38
506a	14.00	16.00	0.87	488	57.9	8.43
506b	14.00	16.00	0.87	488	61.8	7.90
701a	21.00	8.00	2.51	568	62.1	9.15
701b	21.00	8.00	2.51	586	59.6	9.83
702a	21.00	12.00	1.75	373	46.0	8.11
702b	21.00	12.00	1.75	435	53.7	8.10

TABLE 8-2 (cont.) — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Elstner-Hognestad slabs						
A1a	10.00	4.63	2.16	366	45.2	8.10
A1b	10.00	4.63	2.16	442	60.5	7.31
A1c	10.00	4.63	2.16	431	64.9	6.64
A1d	10.00	4.63	2.16	425	73.1	5.81
A1e	10.00	4.63	2.16	431	54.2	7.95
A2a	10.00	4.50	2.22	416	44.5	9.35
A2b	10.00	4.50	2.22	500	53.2	9.40
A2c	10.00	4.50	2.22	583	73.7	7.91
A7b	10.00	4.50	2.22	637	63.6	10.01
A3a	10.00	4.50	2.22	444	43.0	10.33
A3b	10.00	4.50	2.22	555	57.3	9.69
A3c	10.00	4.50	2.22	665	62.0	10.73
A3d	10.00	4.50	2.22	682	70.8	9.63
A4	14.00	4.63	3.03	347	61.6	5.63
A5	14.00	4.50	3.11	475	63.5	7.48
A6	14.00	4.50	3.11	444	60.2	7.38
A7	10.00	4.50	2.22	500	64.3	7.78
A8	14.00	4.50	3.11	388	56.4	6.88
A11	14.00	4.50	3.11	472	61.3	7.70
A12	14.00	4.50	3.11	472	64.2	7.35
B9	10.00	4.50	2.22	629	79.8	7.88
B11	10.00	4.50	2.22	410	44.3	9.26
B14	10.00	4.50	2.22	721	85.6	8.42
Graf slabs						
1362	10.80	10.66	1.11	521	47.6	10.95
1375	10.80	18.62	0.64	413	47.5	8.69

Slab No.	r, in.	d in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Forsell-Holmberg slabs						
1	4.34	3.98	1.09	597	43.6	13.70
2	4.34	4.37	.99	524	43.6	12.09
3	4.34	4.17	1.04	533	43.6	12.23
4	4.34	4.33	1.00	528	43.6	12.11
5	4.34	4.37	.99	588	43.6	13.49
6	4.34	4.21	1.03	566	43.6	12.98
7	4.34	4.17	1.04	583	43.6	13.37
8	0.00	4.37	0.00	∞	52.1	∞
9	0.00	4.21	0.00	∞	52.1	∞
10	10.90†	4.09	2.67	401	52.1	7.70
11	4.34‡	4.42	0.98	411	52.1	7.89
12	4.34‡	4.26	1.02	406	52.1	7.79
14	10.00§	4.26	2.35	540	49.6	10.89
15	4.34	4.33	1.00	528	49.6	10.64
16	4.34	4.26	1.02	586	49.6	11.81
17	4.34‡	4.26	1.02	447	49.6	9.01
18	4.34	4.42	0.98	554	49.6	11.17
19	4.34	4.37	0.99	480	57.1	8.40
20	10.00§	4.33	2.31	658	57.1	11.52
21	10.00§	3.94	2.54	698	57.1	12.22
22	4.34	4.26	1.02	613	57.1	10.73
Scordelis-Lin-May slabs						
S1	13.00	4.25	3.04	474	53.0	8.94
S2	13.00	4.25	3.04	492	63.7	7.74
S4	13.00	2.63	4.94	583	62.9	9.27
S5	13.00	3.00	4.44	383	54.2	7.09
S6	13.00	3.63	3.58	415	68.5	6.06

TABLE 8-2 (cont.) — RELATIONSHIP BETWEEN $v_u/\sqrt{f_c'}$ AND r/d

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Scordelis-Lin-May slabs (cont.)						
S7	13.00	4.38	2.97	532	53.7	9.91
S8	13.00	3.63	3.58	527	66.0	7.98
S9	16.00	3.63	4.41	450	66.2	6.80
S10	16.00	3.63	4.41	506	68.3	7.41
S11	13.00	7.63	1.70	567	71.6	7.92
S12	13.00	5.63	2.31	584	70.1	8.33
S13	13.00	3.63	3.58	577	72.3	7.98
S14	13.00	5.63	2.31	572	69.3	8.25
S15	13.00	3.63	3.58	635	71.5	8.88
Moe's slabs						
R-1	††	4.50	2.67	404	63.2	6.39
R-2	6.00	4.50	1.33	648	62.0	10.45
S5-60	8.00	4.50	1.78	533	56.7	9.40
S6-60	10.00‡‡	4.50	2.22	399	55.8	7.15
S7-60	12.00‡‡	4.50	2.67	437	57.8	7.56
S5-70	8.00	4.50	1.78	590	57.8	10.21
S6-70	10.00‡‡	4.50	2.22	472	59.3	7.96
Elstner-Hognestad slabs (concentrated tensile reinforcement)						
A-9	10.00	4.50	2.22	555	65.8	8.43
A-10	14.00	4.50	3.12	435	65.6	6.63

† Rectangular load area $b = 25.60$ in.
‡ Two load points $b = 8r$.
§ Line load $b = 2r$.
†† Column size 6 x 18 in.
‡‡ Square steel plate.
§§ No slab action.

Slab No.	r, in.	d, in.	$\frac{r}{d}$	v_u test, psi	$\sqrt{f_c'}$ psi	$\frac{v_u}{\sqrt{f_c'}}$
Moe's slabs (concentrated tensile reinforcement)						
S1-60	10.00	4.50	2.22	486	58.1	8.36
S2-60	10.00	4.50	2.22	444	56.6	7.84
S3-60	10.00	4.50	2.22	453	57.3	7.91
S4-60	10.00	4.50	2.22	416	58.8	7.09
S1-70	10.00	4.50	2.22	489	59.6	8.20
S3-70	10.00	4.50	2.22	472	60.7	7.78
S4-70	10.00	4.50	2.22	465	71.4	6.51
S4A-70	10.00	4.50	2.22	388	54.5	7.12
Moe slabs (slabs with openings)						
H-1	10.00	4.50	2.22	462	61.5	7.51
H-2	10.00	4.50	2.22	473	60.2	7.86
H-3	10.00	4.50	2.22	544	58.6	9.28
H-4	10.00	4.50	2.22	486	61.1	7.95
H-5	10.00	4.50	2.22	505	60.2	8.39
H-6	10.00	4.50	2.22	612	64.2	9.53
H-7	10.00	4.50	2.22	432	60.4	7.15
H-8	10.00	4.50	2.22	486	63.8	7.62
H-9	10.00	4.50	2.22	431	59.1	7.29
H-10	10.00	4.50	2.00	417	60.1	6.94
H-11	10.00	4.50	2.22	411	61.5	6.68
H-12	10.00	4.50	∞§§	335	66.1	5.07
H-13	10.00	4.50	∞§§	251	59.7	4.20
H-14	10.00	4.50	2.22	360	61.6	Flexure
H-15	10.00	4.50	2.22	473	58.2	8.13

It is apparent that Eq. (8-14) and (8-15) satisfy the previously stated conditions with $v_u = 4\sqrt{f_c'}$ for r/d equal to infinity and $V_u = 16d^2\sqrt{f_c'}$ for r/d equal to zero.

For practical design purposes, Committee 326 feels that slabs with very large r/d ratios should also be checked for action as a beam, in which case the recommendations of Chapters 5, 6, and 7 are applicable.

804—Comparison with procedures of 1956 ACI Building Code

The proposed concept of shear action is based on the premises that the shear area is the vertical section which follows the periphery at the edge of the loaded area, and that the ultimate shear stress is a function of $\sqrt{f_c'}$ and r/d. The concept of the design procedure in the 1956 ACI Code is different; the shear area is the vertical section which follows the periphery located a distance d beyond the edge of the loaded area, and the allowable shear stress is proportional to f_c' with maximum values of 100 and 75 psi for slabs and footings, respectively.

For purposes of comparison it is assumed that the maximum allowable shear stress by the 1956 Code is 0.03 f_c' and that a safety factor of 2.0 is used. In terms of ultimate shear load, the 1956 Code procedure is: $V_u/bjd = 0.06\ f_c'$. For square columns, $b' = 4(r + 2d)$. Thus, the comparable ultimate shear stress based on a section at the edge of the loaded area is, for $j = 7/8$

$$\frac{v_u}{\sqrt{f_c'}} = \frac{V_u}{4\,rd\sqrt{f_c'}} = 0.0525\sqrt{f_c'}\left(2\frac{d}{r} + 1\right) \quad \text{(8-16)}$$

The 1956 Code requirements are compared with the proposed design equations in Fig. 8-2 and 8-3. In Fig. 8-2, Eq. (8-16) represents a family of hyperbolas having $\sqrt{f_c'}$ as a variable, while Eq. (8-16) is a family of straight lines in Fig. 8-3. The similarity between the 1956 Code design equation and the proposed equations is surprising in view of the fact that the basic concepts of the two procedures appear to be radically diferent. When the 1956 Code design criterion is written in the form of Eq. (8-16), however, it is seen that the variable r/d has been taken into account by assuming the shear area to be located at a distance d beyond the edge of the loaded area. Thus, the variable r/d may be accounted for either as a variable in expressing the shear stress v_u, or by the choice of the location of the shear area.

The 1956 Code procedure may be expressed in terms of the proposed concept as follows:

> The shear area shall be the vertical section which follows the periphery at the edge of the loaded area, and the ultimate unit shear stress shall be a function of f_c' and r/d.

$$v_u = \frac{V_u}{bd} = 0.0525\,f_c'\,(2\,d/r + 1)$$

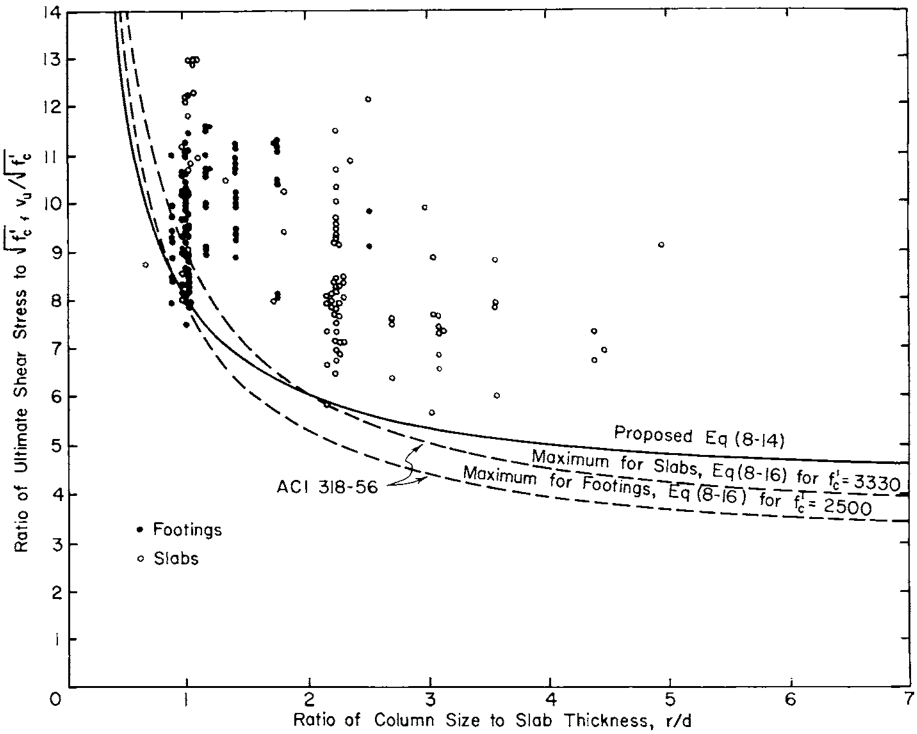


Fig. 8-2—Comparison of Eq. (8-14) to 1956 ACI Building Code

Similarly, the proposed design procedure may be expressed in terms of the 1956 Code concept:

> The shear area shall be the vertical section which follows the periphery at a distance d/2 beyond the edge of the loaded area, and the ultimate shear stress shall be

$$v_u = \frac{V_u}{bd} = 4.0\sqrt{f_c'}$$

Although the two concepts are different, the selection of one in preference to the other becomes a matter of personal choice. The proposed concept follows the line of reasoning adopted by laboratory research; it approaches the basic philosophy which explains the effect of slab action on shear strength. On the other hand, the 1956 Code concept is universally understood and accepted in everyday design practice.

The proposed design procedure offers several advantages over the 1956 Code procedure. In the region of normal flat slab or flat plate design they are more liberal than the 1956 procedures, but they are proven to be safe by laboratory tests. Although it would appear that the same re-

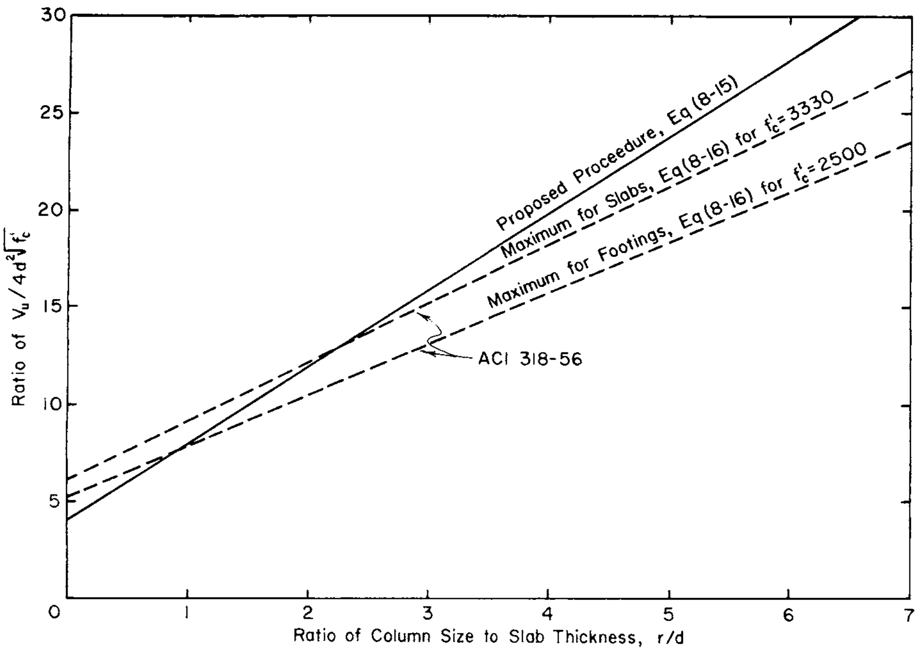


Fig. 8-3—Comparison of Eq. (8-15) to 1956 Building Code

sults may be achieved by increasing the ultimate shear stress of the present procedures, to do so would endanger safety at the extremes in r/d values. The proposed procedure also brings shear strength under two-way slab action in line with the general thoughts regarding the mechanisms of shear and diagonal tension. Furthermore, j is eliminated; and expressing ultimate shear stress in terms of $\sqrt{f_c'}$ facilitates effective use of concrete strengths exceeding 3000 psi.

There is merit, however, in expressing the proposed design procedure in terms of the 1956 Code concept. The proposed method requires expressing the ultimate shear stress as a function of r/d. For a loaded area of other shape than square, there may be doubt regarding the correct value of r to be used. On the other hand, by assuming the ultimate shear stress as equal to $4.0\sqrt{f_c'}$ and defining the critical section as that which is located at a distance $d/2$ from the periphery of the loaded area, regardless of its shape, the question of interpretation regarding the correct value of r does not arise. Furthermore, this gives a simple method of handling the case of openings in the vicinity of the loaded area.

The ultimate shear load capacity V_u can, therefore, be computed by

$$V_u = v_u bd \quad \text{(8-17)}$$

where

$$v_u = 4.0\sqrt{f_c'} \qquad (8\text{-}18)$$

and

b = the periphery of a pseudocritical section located at a distance $d/2$ from the periphery of the loaded area.

805—Concentration of reinforcement over a column

The 1956 ACI Building Code permits an increase in allowable shear stress if the tensile reinforcement of the column strip is concentrated over the periphery of the shear area. A thorough search of the technical literature failed to reveal the origin of, or the logic behind, this requirement. It was probably not based on laboratory tests.

Recent tests by Elstner and Hognestad and by Moe indicated no increase in shear strength due to concentration of tensile reinforcement through the shear area. However, Moe pointed out the following advantages from concentrating tensile reinforcement in the vicinity of the loaded area:[20]

1. Concentration increased the stiffness of the slabs; center deflections decreased as the amount of concentration increased.

2. Concentration reduced the stresses in the flexural tension reinforcement in the vicinity of the column and thereby raised the load at which first yielding took place.

3. Even with heavy concentrations, bond failure or splitting failure was not detected. The concentration reduced the violence of the shear failure.

These tests indicated advantages of concentrating tension reinforcement in the vicinity of the column, but the advantages were realized in the flexural behavior of the slabs, not in their shear behavior.

Because of its advantages in flexure, concentration of tensile reinforcement in the vicinity of the loaded area should be encouraged. However, Committee 326 feels that such encouragement should not be tied to the design requirements for shear.

806—Slabs with openings

Moe[20] reported 15 test specimens having different patterns of openings adjacent to the column. Fourteen of the specimens failed in shear. The tests showed that the ultimate shear strength of the slabs is affected by the size and location of the openings with respect to: the loaded area, the size of the loaded area, and the thickness of the slab.

The effect of the openings on the shear strength of the slabs can be accounted for by reducing the perimeter of the pseudocritical section which is assumed to be at a distance of $d/2$ from the loaded area. This

TABLE 8-3 — EFFECT OF OPENINGS[20]

Slab No.	Periphery, b_o†		P_{test}, kips	$\sqrt{f_c'}$ psi	P_{test} corrected to $\sqrt{f_c'} = 61.5$		Test‡/Calc
	in.	Percent of H-1			kips	percent of H-1	
H-1	58.00	100	83.5	61.5	83.5	100	1.00
H-2	50.75	87.5	74.0	60.2	75.5	90.5	1.03
H-3	43.50	75.0	73.0	58.6	76.5	91.7	1.22
H-4	43.50	75.0	65.1	61.1	65.5	78.5	1.05
H-5	36.25	63.4	56.1	60.2	57.4	68.7	1.08
H-6	29.00	50.0	55.2	64.2	53.0	63.5	1.27
H-7	52.88	91.2	70.1	60.5	71.3	85.5	0.94
H-8	47.76	82.5	70.1	63.8	67.5	80.9	0.98
H-9	52.80	91.0	70.3	59.1	73.0	87.5	0.96
H-10	54.00	93.0	75.1	60.2	76.8	92.0	0.99
H-11	55.20	95.2	76.1	61.5	76.1	91.4	0.96
H-12	29.00	50.0	60.4	63.5	58.5	69.0	1.38
H-13	19.36	33.4	45.1	59.8	46.4	55.6	1.66

† Periphery of critical section b_o, was calculated at $d/2$ from loaded area subtracting the projection of openings in accordance with the procedures recommended in Section 806.

‡ Test/Calc = ratio of percent reduction in corrected measured ultimate shear load, P_{test}, to calculated percent reduction in periphery b_o; in both cases percent reduction with respect to Slab H-1 which had no openings.

reduction in perimeter length depends on the size and location of the openings and the r/d ratio.

The ultimate shear load capacity V_u can be evaluated from Eq. (8-17) and (8-18) as

$$V_u = v_u b_o d = 4.0 \sqrt{f_c'} b_o d \quad \text{(8-17) (8-18)a}$$

reduction in perimeter length depends on the size and location of the pseudocritical section located at a distance $d/2$ from the periphery of the loaded area. Several examples of proposed methods for computation of reduced periphery b_o are shown in Fig. 8-4.

1. For an opening whose closest edge is located less than $d/2$ from the loaded area, it is proposed that the reduced perimeter be the length of the original pseudocritical section minus the radial projection of the opening on the pseudocritical section. The radial lines should be drawn from the centroid of the loaded area to the edges of the opening so that the radial lines lie completely outside the opening as shown in Fig. 8-4a. If there are several openings, the sum of the radial projections should be subtracted from the perimeter of the original pseudocritical section.

2. For openings whose closest edges are more than $d/2$ but less than $2d$ from the loaded area, the reduced perimeter should be taken as the smaller of the two given by the following criteria:

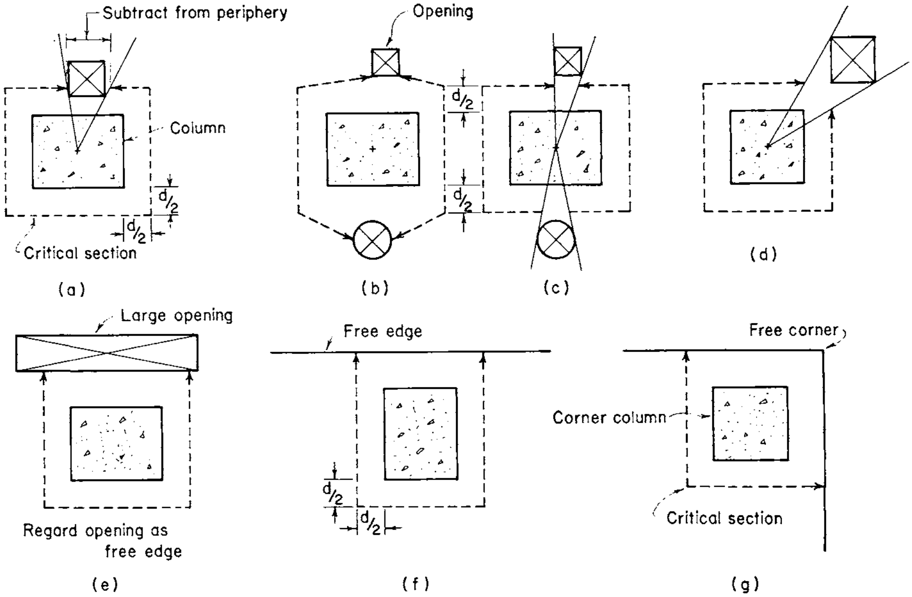


Fig. 8-4—Effect of openings and free edges

(a) The shortest of all possible sections lying not less than $d/2$ from the loaded area as shown in Fig. 8-4b.

(b) The original pseudocritical section minus the sum of the radial projections of the openings as shown in Fig. 8-4c.

3. For openings near the corners of the original critical section, such as the case shown in Fig. 8-4d, Criterion (a) gives no reduction. Criterion (b) probably overestimates the reduction.

4. For the effect of openings whose closest edges are more than $2d$ from the loaded area, only Criterion (a) and the original unreduced critical section need be investigated.

5. Openings that are large compared with the dimensions of the critical section, such as that shown in Fig. 8-4e, should be treated as free edges or corners as described below.

6. The shear capacity of the slab in the vicinity of free edges or corners, as shown in Figs. 8-4f and g, should be evaluated by applying Criterion (a). Particularly when the free edge is located at some distance from the column or loaded area, the original critical section should also be investigated.

7. If several openings are close together, Criteria (a) and (b) may be used provided the distance between openings parallel to the critical section is sufficient to maintain two-way slab action. The

TABLE 8-4 — TRANSFER OF AXIAL LOAD AND MOMENTS FROM COLUMN TO SLAB

Slab No.	r, in.	d, in.	e, in.	$\sqrt{f_c'}$ psi	ϕ_o	V_o,† kips	V_{calc},‡ kips	V_{test}, kips	$\frac{V_{test}}{V_{calc}}$
				Moe slabs[20]					
M1A	12.00	4.50	0.00	55.0	0.691	99.5	99.5	97.3	0.978
M2A	12.00	4.50	7.30	47.4	0.631	84.4	52.5	47.8	0.912
M4A	12.00	4.50	17.10	50.6	0.671	92.8	38.3	32.3	0.844
M2	12.00	4.50	7.70	61.1	0.694	104.2	63.5	65.7	1.035
M3	12.00	4.50	13.30	57.4	0.701	103.1	48.9	46.6	0.954
M4	12.00	4.50	17.20	59.8	0.711	106.5	43.8	29.6	§
M5	12.00	4.50	24.20	62.5	0.726	110.3	36.6	22.7	§
M6	10.00	4.50	6.62	64.2	0.882	81.3	48.9	53.8	1.100
M7	10.00	4.50	2.40	60.2	0.916	83.6	67.4	70.0	1.039
M8	10.00	4.50	17.20	59.7	0.909	83.2	30.6	33.6	1.100
M9	10.00	4.50	5.00	62.1	0.897	81.4	54.3	60.0	1.105
M10	10.00	4.50	12.12	57.8	0.877	78.7	35.6	40.0	1.125

† Calculated ultimate capacity neglecting shear stress due to eccentric loading.
‡ Calculated by Moe's assumption that one-third of the column moment is transferred to slab by vertical shear.
§ Failed in negative bending near the column.

required distance is a function of slab depth, size of openings and other parameters. In extreme cases, a plurality of openings may create a free edge condition.

These criteria were applied to Moe's tests on slabs with openings as shown in Table 8-3. Slab H-1, which had no openings, was considered as the standard, and all slabs with openings were compared relative to this standard. As seen in Table 8-3, the use of the reduced length of the pseudocritical perimeter accurately predicts the reduction in capacity caused by the presence of openings. The strengths of Slab H-6 with openings adjacent to all four faces of the loaded area, and Slabs H-12 and H-13 with openings at all four corners are predicted conservatively. This is desirable because these latter three cases are extremes that should be avoided in practical design applications.

When relatively minor openings are present, it is safe to assume that the shear stress is uniformly distributed over the reduced critical section located at a distance $d/2$ from the column or loaded area. This ultimate stress v_u should then not exceed $4\sqrt{f_c'}$. When large openings, a plurality of openings, or free edges are present, however, it becomes necessary to consider transfer of bending moment. This leads to a non-uniform distribution of shear stress as described in Section 807.

807—Transfer of moment between columns and slabs

Only limited information is available regarding the shear strength of slabs near columns when both axial load and moment are transferred. Moment is transferred between column and slab by flexural moments,

by torsional moments and by vertical shear as shown in Fig. 8-5. No experimental method has been found so far of directly measuring their individual contributions to the total transferred moment. The three quantities can be inter-related by mathematical analyses, but such analyses must be based on simplifying assumptions which may or may not be realistic.

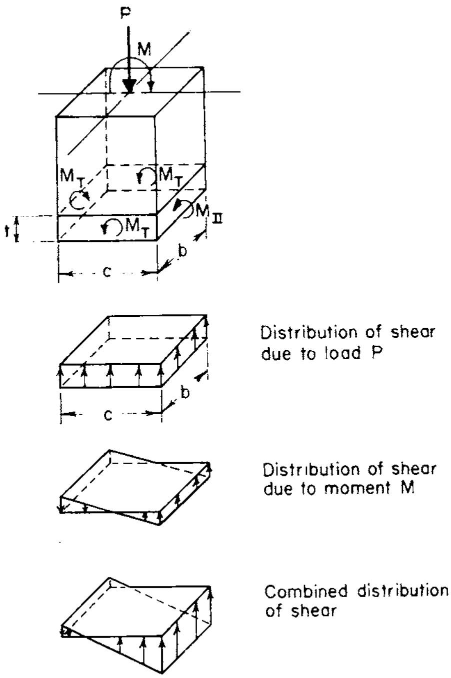


Fig. 8-5—Transfer of moment

Moe[30] reported tests of 12 specimens which had eccentricity of column load as the primary variable. His specimens were square slabs having a centrally located column stub. Eccentricity was varied from 0 to 24 in. on 12 and 10 in. square columns. In his analysis of the tests, Moe did not attempt to separate the transfer moment into its three components. He assumed that the vertical shear stresses were constant across the critical planes perpendicular to the plane of symmetry as shown in Fig. 8-5; and they were assumed to vary linearly along the other two critical planes. Secondly, he assumed that failure occurred when the maximum shear stress reached a value equal to the ultimate shear strength of the same slab loaded with zero eccentricity. Based on these assumptions, Moe worked backward from his test data and found that approximately one-third of the total moment *M* was transferred by vertical shear stresses. This finding is, of course, limited to the type and size of specimen used by Moe. The results of Moe's tests and analysis are summarized in Table 8-4.

In a study which is still in progress, Hanson[31] investigated the shear and moment transfer between slabs and columns by testing ten 3 in. thick slabs with 6 in. square columns. The rectangular slabs were 48 in. wide and 84 in. long, with the column centrally located. They were tested with line loads applied to the slab 36 in. from the column center to create various combinations of shear and moment. Five tests by Frederick and Pollauf[32] which were similar to Hanson's are summarized together with Hanson's tests in Table 8-5. In this group of 15 tests, the range of eccentricity of load was from zero to near infinity.

Calculation of ultimate shear strength for 25 tests of these three investigations at various distances x outside the column are summarized in Table 8-6. In these calculations, which follow the general form developed for interior columns by Di Stasio and van Buren[33] rather than the methods used by Moe, the shear stresses v_1 and v_2 in Fig. 8-6 are

$$\left.\begin{aligned} v_1 &= \frac{8t}{7d}\left[\frac{V}{A_c} + \frac{KM}{J_c}\left(\frac{c}{2}\right)\right] \\ v_2 &= \frac{8t}{7d}\left[\frac{V}{A_c} - \frac{KM}{J_c}\left(\frac{c}{2}\right)\right] \end{aligned}\right\} \qquad (8\text{-}19)$$

where

$$A_c = 2(c + b)t \qquad (8\text{-}20)$$

is the area subject to direct shear and

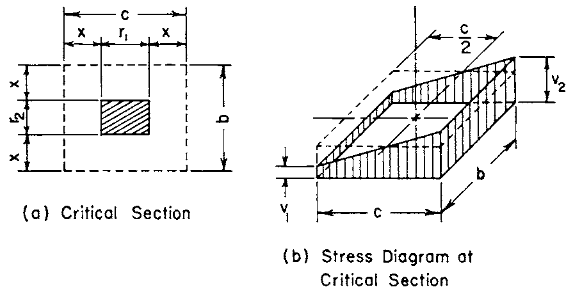
$$J_c = \frac{2\,tc^3}{12} + \frac{2\,ct^3}{12} + 2\,bt\left(\frac{c}{2}\right)^2 \qquad (8\text{-}21)$$

TABLE 8-5 — TRANSFER OF AXIAL LOAD AND MOMENTS FROM COLUMN† TO SLAB‡

Specimen No.	Voids in slab along side of columns	d, in.	$\sqrt{f_c'}$ psi	Transferred	
				V_{test}, kips	M_{test}, in.-kips
Hanson slabs[31]					
1	None	2.44	66.2	1.29	197.6
4	B§	2.44	73.2	0.92	213.3
5	C††	2.44	71.2	1.14	139.6
2	None	2.44	69.4	6.04	181.4
6	B	2.44	68.9	5.88	175.9
7	C	2.44	71.8	4.30	118.9
13	Column at slab edge	2.44	67.2	2.71	87.9
8	None	2.44	67.3	1.08	215.9
9	B	2.44	69.6	1.08	210.7
10	C	2.44	71.1	1.04	150.6
Frederick and Pollauf slabs[32]					
1	None	2.44	54.7	6.09	175.4
2	None	2.44	41.8	7.62	176.0
3	None	1.94	49.7	5.06	106.0
5	None	2.44	49.5	7.07	160.4
6	None	2.44	50.9	7.07	160.4

† All columns 6 x 6 in.
‡ All slabs 3 in. thick except Specimen 3 (t = 2.5 in.).
§ Voids along sides of column parallel to axis of stress symmetry.
†† Voids along sides of column perpendicular to axis of stress symmetry; all voids were 6 x 1 in.

Fig. 8-6 — Shear stress at distance x



is the polar moment of inertia of the surface described by the critical section passing through the slab thickness as shown in Fig. 8-6.

An increase in A_c and J_c may be made to account for dowel action of the steel crossing the area by multiplying the individual terms of Eq. (8-20) and (8-21) by $[1 + (n - 1)\, p]$.

The factor K in Eq. (8-19) is a reduction factor on the total moment transferred M, to obtain the moment transferred by torsional shear stress.

It is shown in Table 8-6 that the best correlation with the test data was obtained when the shear stresses were calculated at a distance $x = d$ outside the column with full dowel action considered. The value $K = 0.487$ gave the best coefficient of variation of 0.121 for the 25 tests available. The average value of the maximum shear stress at ultimate strength was $4\sqrt{f_c'}$ psi.

To be consistent the design procedures developed earlier in this chapter, Eq. (8-19), (8-20) and (8-21) can be written as

$$v_u = \frac{V}{A_o} + \frac{KM}{J_c}\left(\frac{c}{2}\right) \quad \text{(8-22)}$$

where

$$A_c = 2(c + b)d = b_o\, d \quad \text{(8-23)}$$

and

$$J_c = \frac{2\, dc^3}{12} + \frac{2\, cd^3}{12} + 2\, bd\left(\frac{c}{2}\right)^2 \quad \text{(8-24)}$$

In Eq. (8-22) to (8-23), the critical section is taken at a distance $x = d/2$ from the face of the loaded area. When the perimeter of the critical section is reduced for exterior columns or when openings are present, Fig. 8-4, the polar moment of inertia should be computed on the basis of the remaining section and taken about the centroidal axis of the reduced perimeter. Furthermore, consistent with other parts of this report, no increase in shear resistance due to dowel action should be made for Eq. (8-23) and (8-24).

Using the results of the 25 tests available, the factor K was re-evaluated for Eq. (8-22), taking into account the reduction of the perimeter of the

TABLE 8-6 — SUMMARY OF CALCULATIONS OF ULTIMATE SHEAR STRESS FOR 25 SPECIMENS WITH MOMENT TRANSFER

Distance outside column, x	Dowel action of steel	Best coefficient, K	Coefficient of variation	Average $\frac{v_{test}}{\sqrt{f_c'}}$	Based on equation
0	Full	0.150	0.290	5.64	
0	None	0.148	0.272	6.62	
$t/2$	Full	0.331	0.143	4.32	(8-19)
$t/2$	None	0.312	0.182	5.04	
d	Full	0.487	0.121	3.99	
d	None	0.437	0.180	4.51	
$d/2$	None	0.200	0.259	4.47	(8-22)

critical section for those slabs which had openings. As shown in Table 8-6, a constant $K = 0.20$ gave a coefficient of variation of 0.26 and an average value of calculated ultimate shear stress, $v_u = 4.47\sqrt{f_c'}$ psi, so that the value $v_u = 4\sqrt{f_c'}$ used in other parts of this report appears to be a safe design value.

The r/d-ratio in the tests considered here was between 2.23 and 3.10. Additional tests are necessary to establish the strength for other values of the r/d-ratio. Until further experimental data are available, these procedures may serve as a general guide to design.

808—Shear reinforcement in slabs

Shear reinforcement transfers shear force across a diagonal tension crack. To accomplish this purpose, the shear reinforcement must be securely anchored at both ends. Generally speaking, anchorage of stirrups or bent-up bars fall into two categories. First, anchorage can be developed by transferring the force in the shear bar to other reinforcement, such as by rigidly attaching stirrups to longitudinal reinforcement or by tightly wrapping stirrups around the longitudinal reinforcement. Secondly, anchorage can be developed by transferring the force from the shear bar to the concrete by bond and bearing. The second category is exemplified by standard hooks on stirrups in which bond is controlled by embedment length and bearing is controlled by specifying minimum bend radii. Anchorage by bond and bearing should, of course, be confined to the compression zone of the concrete.

In beams of normal size, anchorage of shear reinforcement is rarely a serious problem. However, in slabs, it is a major problem which becomes more acute as the thickness of the slab diminishes. Consider a slab 6 in. thick. With normal percentages of tensile reinforcement, the depth of the compression zone will be 1 in. or less. Since the reinforcement must have at least a ¾-in. cover, it becomes impossible to anchor the shear reinforcement by bond within the compression zone of the slab.

Similarly, the bending of diagonal bars or stirrups presents difficulties in slabs. Generally speaking, the requirements for minimum radii of bends are dependent on the bending properties of the steel. However, minimum radii serve another purpose; i.e., limiting the direct compression or bearing on the concrete under the bend of the reinforcing bar. The bearing stresses on the concrete under the bend will depend on the radius of the bend and the load carried by the bar at the beginning of the bend. In beams, bends of minimum radii are easily located well above the neutral axis. A part of the load carried by the bar at a diagonal crack is transferred to the concrete by means of bond over the bar length from the diagonal crack to the beginning of the bend. Therefore, bearing stresses under bar bends are rarely a problem in beams.

However, two factors make bearing stresses an acute problem in slabs. Because of space limitations it is often difficult to have bends of minimum radii. Secondly, the diagonal cracks will cross the bar much closer to the beginning of the bend so that little of the load in the bar will be taken out by bond between the diagonal crack and the beginning of the bend. Both factors cause increased bearing stresses on the concrete under the bar bend.

Tests by Elstner and Hognestad[17] confirmed the difficulties which can be encountered with shear reinforcement in their slabs. Inability to anchor shear reinforcement within the compression zone caused the shear failure plane to pass around, rather than through, the shear reinforcement. Likewise, bearing failures of the concrete under shear bar bends were observed.

The test specimens of Elstner and Hognestad had effective depths of 4.5 in. Graf[5] reported six tests of slabs with shear reinforcement. Graf's slabs had effective depths of 10.7 and 18.7 in. Data from the 14 tests are presented in Table 8-7. The data are meager; the variables have not been fully explored. Unfortunately, the thin slabs of Elstner and Hognestad were limited to light shear reinforcement while the thick slabs of Graf were limited to heavy shear reinforcement. Therefore, it is not possible to determine the effect of poor anchorage in thin slabs since the two sources of data are not comparable on the basis of slab thickness being the only variable.

However, Table 8-7 does present interesting information. In all slabs except one, shear reinforcement increased the load capacity, and in all cases except one the ultimate load capacity was greater than either the load capacity of the concrete alone or the load capacity of the shear reinforcement alone. However, in no case was the ultimate load capacity of the slab equal to the sum of the load capacities of concrete and the shear reinforcement.

The conclusion may be drawn that the shear reinforcement was not fully effective, although this conclusion cannot be firmly supported by

TABLE 8-7 — SLABS WITH SHEAR REINFORCEMENT

Slab No.	r, in.	d, in.	$\sqrt{f_c'}$ psi	ϕ†	v_u,‡ psi	Krf_y, psi	v_u' test, psi	$\frac{v_u'}{v_u}$	$\frac{Krf_y}{v_u}$
				Elstner-Hognestad slabs[17]					
B-3	10.00	4.50	43.9	1.040	309	229	358	1.159	0.741
B-5	10.00	4.50	45.6	0.614	423	211	472	1.116	0.499
B-6	10.00	4.50	49.6	0.768	419	422	584	1.394	1.007
B-10	10.00	4.50	82.0	0.832	667	422	666	.999	0.633
B-12	10.00	4.50	81.5	0.856	653	574	982	1.540	0.879
B-15	10.00	4.50	84.2	0.743	724	638	860	1.187	0.881
B-16	10.00	4.50	81.0	0.798	673	854	932	1.385	1.269
B-17	10.00	4.50	45.8	0.888	359	229	445	1.240	0.638
				Graf slabs[5]					
1355	7.90	10.70	47.0	0.980	425	681	802	1.887	1.602
1356	7.90	10.80	47.0	1.042	410	669	846	2.063	1.632
1376	7.90	18.70	48.7	0.986	454	704	858	1.890	1.551
1377	7.90	18.70	47.0	0.966	443	704	840	1.896	1.589
1361	11.80	10.70	48.7	0.965	425	620	768	1.807	1.459
1363	11.80	18.50	48.7	0.852	479	617	778	1.624	1.288
				Moe slab with shearhead[20]					
S8-60	8.00	4.50	57.8	0.980	454	222	573	1.262	0.489

† $\phi = V_{test}/V_{flex}$
‡ v_u = shear stress calculated by Eq. (8-11), and neglecting the shear reinforcement.

test data. In the thin slabs there is little doubt that poor anchorage limited the effectiveness of the shear reinforcement. In the thick slabs the heavy shear reinforcement may have caused shear-compression failures before the shear reinforcement was fully effective.

In view of these circumstances, it is not possible at this time to recommend detailed design procedures for shear reinforcement in slabs. However, the following points can be emphasized:

1. Because of difficulties in anchorage and bending, shear reinforcement should not be permitted in slabs less than 10 in. thick.

2. Although shear reinforcement is beneficial, the required shear steel area may be abnormally large to increase load capacity even a small amount.

3. Restrictions on the use of shear reinforcement do not necessarily penalize flat plate design. The proposed design procedure is in many cases more liberal than that of the 1956 ACI Code. Furthermore, the proposed procedure permits higher shear stresses for concrete strengths exceeding 3000 psi. Therefore, it would seem more safe and practical, and possibly more economical, to increase shear capacity by using higher strength concrete rather than by using shear reinforcement.

809—Recommendations for design

Experimental investigations have indicated that the ultimate shear strength of slabs is dependent on three major variables: concrete strength $\sqrt{f_c'}$; ratio of column width to slab thickness r/d; and ratio of shear capacity to flexural capacity ϕ_o. In normal design practice, the shear capacity should be equal to or slightly greater than the flexural capacity. Therefore, ϕ_o was taken as unity in the development of design recommendations.

The variable r/d can be taken into account in two ways. First, the ultimate shear stress v_u can be expressed as a function of r/d in accordance with Eq. (8-14). The ultimate shear load capacity V_u can then be computed by Eq. (8-15), in which the critical section is the periphery around the loaded area. Secondly, the variable r/d can be taken into account by choosing a pseudocritical section which is located a distance $d/2$ from the periphery of the loaded area. The corresponding ultimate shear stress on this pseudocritical section is independent of r/d and is equal to $4.0\sqrt{f_c'}$ in accordance with Eq. (8-18). It has been pointed out that the latter method seems preferable because of its simplicity, especially for irregularly shaped loaded areas, and when openings, free edges or free corners are present in the vicinity of the loaded area. Furthermore, the second method involves a familiar concept similar to the method used in the 1956 ACI Building Code.

The following design recommendations are based on the previous discussion in this chapter:

(a) The shear strength of slabs and footings near a concentrated load or reaction is governed by the more severe of two conditions:

1. The footing or slab may act essentially as a wide beam with a potential diagonal crack extending in a plane across the entire width. This case shall be considered in accord with the recommendations made in Chapters 5, 6 and 7 of this report.

2. Two-way slab action may exist, with potential diagonal cracking along the surface of a truncated cone or pyramid around the concentrated load or reaction. This case shall be considered as described under Recommendations (b) through (j).

(b) Although the proposed concept of shear strength, when slab action is present, is based on the premises that the shear area is the vertical section which follows the periphery at the edge of the loaded area, and that the ultimate shear stress is a function of $\sqrt{f_c'}$ and r/d, an approximately equal shear strength can be evaluated by assuming that the shear area is a pseudocritical vertical section located at a distance $d/2$ from the periphery of the loaded area, and that the ultimate shear stress is a function of $\sqrt{f_c'}$ only.

It is therefore recommended that shear strength shall be computed by

$$v_u = \frac{V_u}{b_o\, d} \leq 4.0 \sqrt{f_c'}$$

where

v_u = permissible ultimate shear stress
f_c' = compressive strength of 6 x 12 in. concrete cylinders
V_u = the ultimate shear force on a pseudocritical section of area $b_o\, d$
b_o = the effective periphery of the pseudocritical section at a distance $d/2$ from the periphery of the loaded area, taking into account the effect of openings, free edges or free corners in the vicinity of the loaded area
d = th effective depth of the slab at the periphery b_o

(c) Openings in slabs, free edges, and corners in the vicinity of the loaded area shall be considered by reducing the periphery of the pseudocritical section in accordance with the recommendations of Section 806. That part of the periphery of the pseudocritical section which is covered by radial projections of openings to the centroid of the loaded area shall be considered ineffective, or the shortest periphery of a critical section shall be used as outlined in Fig. 8-4.

(d) Flexural reinforcement shall be provided along the edges of all openings and extend as required for anchorage in both directions beyond the openings. However, all flexural reinforcement which would normally pass through the loaded area in slabs without openings, must be so rearranged that it continues to pass through the loaded area when openings are present.

(e) If the effective depth of the slab is less than 10 in., shear reinforcement consisting of bars, rods or wires shall not be considered effective.

(f) If the effective depth of the slab is greater than 10 in., shear reinforcement shall be permitted to carry the excess shear as described in Chapter 6, but the shear reinforcement shall be considered 50 percent effective.

(g) Concentration of flexural reinforcement in a slab over a column or column capital should be encouraged in flexural design, but the permissible ultimate shear stress shall not be increased because of concentration of reinforcement.

(h) If moment is transferred at a slab to column connection, it shall be assumed that, due to torsion, the vertical shear stresses are constant across the pseudocritical sections perpendicular to the plane of symmetry (parallel to the axis of torsion) and vary linearly on the other two pseudocritical sections parallel to the plane of

symmetry (perpendicular to the axis of torsion.) The vertical shear stresses due to the total shear load shall be assumed uniformly distributed over the entire pseudocritical area $b_o d$. It shall further be assumed that the law of superposition applies so that the shear strength shall be computed by

$$v_u = \frac{V_u}{b_o d} + \frac{KM}{J_c}\left(\frac{c}{2}\right) \leq 4.0\sqrt{f_c'}$$

where

M = the total joint moment on the pseudocritical peripheral section about its centroid

J_c = the polar moment of inertia of th pseudocritical peripheral section about its centroid equals $dc^3/6 + cd^3/6 + 2\,bd(c/2)^2$ for a critical section without openings or free edges

c = the side of the pseudocritical section perpendicular to the axis of torsion

b = the side of the pseudocritical section parallel to the axis of torsion

K = a reduction factor on the total moment to obtain the moment transferred by torsional shear stress, found to be 0.2 on the basis of the limited test data available, but may approach zero or take values greater than 0.2 under other conditions.

(i) It should be noted that no design recommendations are made by Committee 326 for lightweight aggregate concrete slabs and footings. Throughout this chapter, reference has been made to ordinary sand and gravel concrete only. The Committee has not considered a series of tests of lightweight aggregate concrete slabs carried out in 1961 at the PCA laboratories.

(j) To apply these recommendations in ultimate strength design, suitable safety provisions must be combined with these recommendations. Development of such safety provisions is considered beyond the scope of this Committee's mission.

810—Test data

The test data considered in this chapter on slabs and footings are presented in condensed form in Tables 8-1 through 8-7. For more detailed information, the reader is referred to the references listed in Section 811.

811—References

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† This publication became available to Committee 326 too late to allow its consideration in development of this chapter.

NOTATION

The frequently used letter symbols of this report are summarized below:

A_c = area of peripheral section in slabs
A_g = gross area of the uncracked section
A_s = area of tensile reinforcement
A_v = area of shear reinforcement
a = spacing of web reinforcement in a direction perpendicular to web reinforcement; also length of shear span; also side length of square footing

b = width of cross section; also perimeter of critical peripheral section; also the side of the peripheral section parallel to the axis of torsion

b' = web width in I- and T-sections

b_o = effective perimeter of peripheral section

C, C', C_1, C_2 = constants

c = the side of the peripheral section perpendicular to the axis of torsion

d = effective depth

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel

e = eccentricity of axial load measured from centroid of tensile reinforcement

F_1, F_2 = constants

f_c' = compressive strength of 6 x 12-in. cylinders

f_c^* = design strength of concrete in flexural compression

f'_{cu} = compressive strength of cubes

f_s = tensile steel stress

f_t = flexural tension stress

f_t' = diagonal tension strength of concrete

f_v = stress in web reinforcement

f_y = yield point of steel

h = total depth of section

I = moment of inertia

J_c = polar moment of inertia of peripheral section about its centroid

jd = internal moment arm

K = (sin α cot θ + cos α) sin α or (sin α + cos α) sin α; also a moment reduction factor

L = length of beam

l_s = shear span in slabs

M = bending moment

$M' = M - N \dfrac{4h - d}{8}$

m = ultimate resisting moment per unit width of slab

N = axial load

n = E_s/E_c

P_u = total load on loaded area

p = ratio of tensile reinforcement = A_s/bd $A_s/b'd$ for I- and T-beams

Q = first moment of part of a cross section

$q_u = \dfrac{A_v f_y \sin \alpha}{\frac{7}{8} bd f_c'}$

R = reaction

r = ratio of web reinforcement = A_v/ab; also side length of loaded area

s = spacing of web reinforcement along longitudinal axis of member

T = force in tension reinforcement

t = total depth of section

V = total shear force

V' = shear force carried by web reinforcement

V_c = shear force carried by concrete
V_{flex} = ultimate shear force for flexural failure
V_u = ultimate shear capacity
V_y = shear force at which web reinforcement yields
v = shear stress
v_c = shear stress allotted to concrete; also ultimate diagonal tension strength of beams without web reinforcement
v_{con} = shear stress in concrete compression zone
v_u = ultimate shear stress; also ultimate shear stress in members with web reinforcement
v_y = V_y/bjd
α = inclination of web reinforcement to longitudinal axis of member
β = $1 - (h/2d) - j$
θ = inclination of diagonal crack to longitudinal axis of member
ϕ_o = V_u/V_{flex}

CLOSING REMARKS BY THE CHAIRMAN

Committee 326 was formed 12 years ago and was given the assignment of "developing methods for designing reinforced concrete members to resist shear and diagonal tension, consistent with the new ultimate strength design methods."

This report consolidates thoughts, engineering judgement, and knowledge gained from engineering practice as well as extensive experimental and analytical investigations into a form believed to be useful to practicing engineers. Furthermore, safe and workable new design procedures are given. The Committee's original mission has been accomplished.

In closing this report, the Chairman wishes to express his personal appreciation to the Committee members for over a decade of active work. Engineers at home and abroad beyond the Committee membership, too numerous to be listed here, have also contributed importantly to this report. Special recognition is due members of ACI Committee 318, Standard Building Code and the European Concrete Committee, as well as Prof. C. W. Thurston of Columbia University.